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**Diego Marques** and **Jonathan Sondow\*** ([jsondow@alumni.princeton.edu](mailto:jsondow@alumni.princeton.edu)). *The Schanuel Subset Conjecture implies the Gelfond Power Tower Conjecture.*

We introduce the *Schanuel Subset Conjecture* (SSC). It states that, if the complex numbers  $\alpha_1, \dots, \alpha_n$  are linearly independent over  $\mathbb{Q}$ , and if the set  $\{\alpha_1, \dots, \alpha_n, e^{\alpha_1}, \dots, e^{\alpha_n}\}$  is  $\overline{\mathbb{Q}}$ -dependent on a subset  $\{\beta_1, \dots, \beta_n\}$ , then  $\beta_1, \dots, \beta_n$  are algebraically independent.

It is easily shown that *Schanuel's Conjecture implies SSC*. Are the two conjectures in fact equivalent?

In a 1934 announcement in *Comptes Rendus*, Gelfond stated a vast generalization of the Gelfond-Schneider Theorem, but he never published a proof. A special case, which we call the *Gelfond Power Tower Conjecture*, asserts that, if  $z = e^\omega$  or  $z = \alpha$ , where  $\omega \neq 0$  and  $\alpha$  are algebraic numbers with  $\alpha$  irrational, then the power towers  $z^z, z^{z^z}, z^{z^{z^z}}, \dots$  are algebraically independent.

Our main result is that, *if the Schanuel Subset Conjecture is true, then the Gelfond Power Tower Conjecture is also true.*

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