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Lawrence A. Harris* (larry@ms.uky.edu), Mathematics Department, University of Kentucky, Lexington, KY 40506. *A Proof of Markov's Theorem for Polynomials on Banach Spaces.*

Our object is to present an independent proof of the extension of V. A. Markov's theorem to the Gâteaux derivatives of continuous polynomials on any real normed linear space. Specifically, we prove that if X and Y are such spaces and if $P : X \rightarrow Y$ is a polynomial of degree at most m satisfying $\|P(x)\| \leq 1$ for all $x \in X$ with $\|x\| \leq 1$, then

$$\begin{aligned}\|\hat{D}^k P(x)\| &\leq T_m^{(k)}(1) && \text{for all } x \in X \text{ with } \|x\| \leq 1 \text{ and} \\ \|\hat{D}^k P(x)\| &\leq T_m^{(k)}(\|x\|) && \text{for all } x \in X \text{ with } \|x\| \geq 1,\end{aligned}$$

where T_m is the Chebyshev polynomial of degree m . An erroneous proof of the first inequality has been given by A. D. Michal in 1954 while a rather daunting proof of a bit more general theorem has been given by V. I. Skalyga in 2005.

We have shown in a previous paper that to prove both inequalities it suffices to prove the second inequality for the case where X is \mathbb{R}^2 with the max norm and $Y = \mathbb{R}$. We do this by extending a simple Lagrange interpolation argument of Rogosinski to two variables and applying a bivariate Christoffel-Darboux formula for the Lagrange interpolation polynomials at the Chebyshev nodes. (Received January 25, 2010)