of the Neumann and Steklov problems on planar domains.
The Pólya conjecture (1954) states that the $k$-th Neumann eigenvalue of a planar domain is bounded above by $4 k \pi$. In this talk I will present a sharp isoperimetric inequality for the second non-zero eigenvalue. This implies the Pólya conjecture for $k=2$. I will also discuss similar results for the spectrum of the Dirichlet-to-Neumann map and for the spectrum of the Laplace-Beltrami operator on spheres. Surprisingly, this extension to spheres is possible only for odd dimensions. (Received January 26, 2010)

