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Richard S Laugesen* (Laugesen@illinois.edu), Department of Mathematics, University of Illinois, Urbana, IL 61801, and Bartlomiej A Siudeja. Sums of Dirichlet eigenvalues: maximizing with rotationally symmetric domains. Preliminary report.

Many sharp inequalities are known for the low Dirichlet eigenvalues of the Laplacian. We present new sharp inequalities for higher eigenvalues, namely for sums of the first n eigenvalues, for each n.

Consider a plane domain D having rotational symmetry of order 3 or greater. We prove that among all domains obtained from D by affine transformation, the scale-invariant eigenvalue sum

$$S_n = (\lambda_1 + \dots + \lambda_n) \frac{A^3}{I}$$

is maximal for D, for each n. Here A denotes the area and I is the moment of inertia of the domain.

Corollaries: S_n is maximal for the equilateral triangle among all triangles. S_n is maximal for the square among all parallelograms. These corollaries extend work of Pólya on the fundamental tone, the case n = 1.

These results suggest a conjecture for convex plane domains: is the normalized eigenvalue sum S_n maximal for the disk? (Received January 25, 2010)