1053-05-203   **Hélène Barcelo** and **Christopher Severs*** (`csevers@msri.org`), The Mathematical Sciences Research Institute, 17 Gauss Way, Berkeley, CA 94720-5070, and **Jacob A. White**. *The discrete fundamental group of the Associahedron and its relation to the type-A cluster algebra.*

The associahedron is a polytope that has received much attention in recent years. It has appeared in the study of areas such as operads, lattice theory, mathematical physics, algebraic geometry and cluster algebras, to name a few. The associahedron is of special interest to combinatorists because the vertices are counted by the Catalan numbers, which leads to many descriptions via Catalan objects such as triangulations of a regular polygon and bracketing of words.

We use the tools of discrete homotopy theory, developed by Laubenbacher and Barcelo, et al. to study the associahedron. We give a combinatorial description for the equivalence classes of the discrete fundamental group in terms of 5-cycles in the 1-skeleton of the associahedron. We then use this description to give a presentation for the discrete fundamental group.

Furthermore, we describe the pentagonal relations in the type-A cluster algebra via the discrete homotopy classes. We show that the homotopy classes correspond exactly to the pentagonal relations and that the classes which generate the abelianization of the discrete fundamental group also generate all pentagonal relations in the cluster algebra. (Received September 03, 2009)