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Vyacheslav Pavlovich Krivokolesko* (antonk@ktk.ru), Krasnoyarsk, 660021, Russia. *An integral representation for linearly convex polyhedra and several combinatorial identities.* Preliminary report.

Using an integral representation from [1] for functions that are holomorphic in a linearly convex polyhedron with a piecewise smooth boundary

$$G = \{z = (z_1, z_2) \in \mathbb{C}^2 : g^1(|z|) = -|z_1| + a < 0, g^2(|z|) = -|z_2| + b < 0, \\ g^3(|z|) = |z_1|^2 + |z_2|^2 - r^2 < 0, r > a, r > b\}. \quad (1)$$

we obtain some combinatorial identities.

Theorem. *For $0 < \alpha < 1$, $0 < \beta < 1$ the following identities hold:*

$$\frac{(s_1 + s_2 + 1)!}{s_1!s_2!} \sum_{m=0}^{s_1} \frac{(-1)^m}{s_2 + m + 1} \binom{s_1}{m} ((1 - \alpha)^{s_2+m+1} - \beta^{s_2+m+1}) \\ \equiv (1 - \alpha)^{s_2+1} \sum_{m=0}^{s_1} \binom{s_2 + m}{m} \alpha^m - \beta^{s_2+1} \sum_{m=0}^{s_1} \binom{s_2 + m}{m} (1 - \beta)^m, \quad (2)$$

$$1 \equiv \frac{(s_1 + s_2 + 1)!}{s_1!s_2!} \sum_{m=0}^{s_1} \frac{(-1)^m}{s_2 + m + 1} \binom{s_1}{m} ((1 - \alpha)^{s_2+m+1} - \beta^{s_2+m+1}) + \\ \alpha^{s_1+1} \sum_{m=0}^{s_2} \binom{s_1 + m}{m} (1 - \alpha)^m + \beta^{s_2+1} \sum_{m=0}^{s_1} \binom{s_2 + m}{m} (1 - \beta)^m. \quad (3)$$

Corollary. For $m = 0, \dots, s_1$ the following identities are valid:

$$\binom{s_2 + m}{s_2} = (-1)^m \frac{(s_1 + s_2 + 1)!}{s_1! s_2!} \sum_{k=m}^{s_1} \frac{(-1)^k}{s_2 + k + 1} \binom{s_1}{k} \binom{k}{m} \quad (4)$$

In the special case $m = 0$ we recover the formula No.45 on page 611 in [2].

Bibliography:

- [1] Krivokolesko V.P. and Tsikh A.K., Integral Representations in Linearly Convex Polyhedra, Siberian Mathematical Journal, Vol. 46, No.3, 579-593 (2005). Sib. Mat. J, Vol. 46, No.3, 579-593 (2005).
- [2] Prudnikov A.P., Brychkov Yu.A., Marichev O.I. Integrals and rows [in Russian], Moskov, Nauka (1981). (Received June 01, 2009)