Generalized Sum-Dominant Sets.
Many of the biggest problems in additive number theory (such as Goldbach's conjecture and Fermat's last theorem) can be recast as understanding the behavior of sums of a set with itself. A sum-dominant set is a finite set $A \subset \mathbb{Z}$ such that $|A+A|>|A-A|$. It was initially believed that the percentage of subsets of $\{0, \ldots, n\}$ that are sum-dominant tends to zero, however, in 2006 Martin and O'Bryant proved a positive percentage are sum-dominant.

We generalize their result to deal with many different ways of taking sums and differences of a set. We first prove that $\left|\epsilon_{1} A+\cdots+\epsilon_{k} A\right|>\left|\delta_{1} A+\cdots+\delta_{k} A\right|$ a positive percent of the time for all nontrivial choices of $\epsilon_{j}, \delta_{j} \in\{-1,1\}$. Previous approaches proved the existence of many such sets given the existence of one; however, no method existed to construct such a set.

Extending this result, we find sets that exhibit different behavior as more sums/differences are taken. For example, we say $A$ is $k$-generational if $A, A+A, \ldots, k A$ are all sum-dominant. Numerical searches were unable to find even a 2-generational set, however, we prove that for any $k$ a positive percentage of sets are $k$-generational, and no set can be $k$-generational for all $k$. (Received September 14, 2011)

