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**Jonathan Sondow\*** (jsondow@alumni.princeton.edu). *Evaluation of infinite products involving Fibonacci and Lucas numbers.*

The *Fibonacci numbers*  $F_k$  are defined by the recurrence  $F_k = F_{k-1} + F_{k-2}$  with  $F_0 = 0$  and  $F_1 = 1$ ; the first few terms are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, . . . . The *Lucas numbers*  $L_k$  satisfy the same recurrence, but with  $L_0 = 2$  and  $L_1 = 1$ ; the sequence begins 2, 1, 3, 4, 7, 11, 18, 29, 47, 76, . . . .

In this talk, I will begin by recalling formulas for  $F_k$  and  $L_k$  in terms of the *golden ratio*  $\varphi = (1 + \sqrt{5})/2$ . Then I will explain how to evaluate the infinite products

$$\prod_{n=1}^{\infty} \left(1 + \frac{1}{F_{2^{n+1}}}\right) = \frac{3}{2} \frac{6}{5} \frac{35}{34} \cdots = \frac{3}{\varphi}, \quad \prod_{n=1}^{\infty} \left(1 + \frac{1}{L_{2^{n+1}}}\right) = \frac{5}{4} \frac{12}{11} \frac{77}{76} \cdots = 3 - \varphi.$$

A preprint is available at <http://arxiv.org/abs/1106.4246>. (Received September 22, 2011)