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**Nadine Amersi, Olivia Beckwith, Steven J Miller and Ryan Ronan\***  
(ronan2@cooper.edu), 32-52 41st Street, Apartment #2A, Queens, NY 11103, and **Jonathan Sondow**. *The Distribution of Generalized Ramanujan Primes*.

In 1845, Bertrand conjectured that for all integers  $x$  greater than or equal to 2, there exists at least one prime in  $(x/2, x]$ . This was proved by Chebyshev in 1860, and then generalized by Ramanujan in 1919, who showed for any integer  $n$  there is a prime  $R_n$  such that  $\pi(x) - \pi(x/2) \geq n$  for all  $x \geq R_n$ . We generalize the interval of interest by introducing a parameter  $c \in (0, 1)$  and defining the  $n$ -th  $c$ -Ramanujan prime  $R_{c,n}$  as the smallest integer such that for all greater integers  $x$ , there are at least  $n$  primes between  $cx$  and  $x$ . Using consequences of strengthened versions of the Prime Number Theorem, we prove the existence of  $R_{c,n}$  for all  $n$  and all  $c$ , that the asymptotic behavior is  $R_{c,n} \sim p_{\frac{n}{1-c}}$  (where  $p_m$  is the  $m$ -th prime), and that the percentage of primes that are  $c$ -Ramanujan converges to  $1 - c$ . We then study finer questions related to their distribution among the primes, and see that the  $c$ -Ramanujan primes display striking behavior, deviating significantly from a probabilistic model based on biased coin flipping. This model is related to the Cramer model, which correctly predicts many properties of primes on large scales but has been shown to fail in some instances on smaller scales. (Received September 19, 2011)