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Karel Hrbacek* (khrbacek@sci.ccny.cuny.edu). *Analysis based on the concept of level.*

This is joint work with R. O'Donovan and O. Lessmann. We propose a framework for analysis that is loosely modeled on the physicists' intuitive but vague concept of scales of magnitude. We take the point of view that every mathematical entity appears at some *level* \mathbf{V} . Levels are *not sets*, but they have some definite properties: For every mathematical object x there is a coarsest level \mathbf{V} where x appears; For every level \mathbf{V} , there exist real numbers h *ultrasmall relative to* \mathbf{V} (i.e., such that $0 \neq |h| < r$ for every $r > 0$ that appears at \mathbf{V}); A number, function or set that is uniquely defined from parameters at some level \mathbf{V} , appears itself at the level \mathbf{V} . These postulates suffice to define and calculate derivatives and integrals in the style of Leibniz. A few additional axioms make possible a fully rigorous development of elementary analysis. Use of ultrasmall numbers dispenses with the epsilon-delta machinery, proofs become simpler, and fundamental results can be proved from first principles without need for the notion of supremum. Epsilon-delta arguments can be introduced naturally when studying estimation. The approach has been classroom-tested in Geneva. (Received September 22, 2011)