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**Judit Kardos\*** ([kardosj@tcnj.edu](mailto:kardosj@tcnj.edu)), TCNJ, Mathematics Department, 2000 Pennington road, Ewing, NJ 08628. *Understanding the infinite*. Preliminary report.

Hegel writes in the Science of Logic that “Mathematics owes its most brilliant successes to ideas contradicting discursive reason.” The idea of the actually infinite is one such idea.

Let us call a subset of the natural numbers,  $S$ , potentially infinite if for any  $n \in S$  there exists an  $m \in S$  satisfying  $m > n$ . A good initial problem that leads students to realize the difference between the actual and the potential infinite is the following simple question: “Can we color the natural numbers using two colors, say red and blue, so that neither the blue set nor the red set contains any infinite arithmetic progressions?” Students frequently come up with the following solution:

**1, 2, 3, 4, 5, 6,** 7, 8, 9, 10, **11, 12, 13, 14, 15,** 16, 17, 18, 19, 20, 21, ...

Clearly, the length of the arithmetic progressions both in the red (***bold***) and in the blue set (*standard*) form a potentially infinite set, but it is easy to prove that the above coloring will allow for no actually infinite arithmetic progression in either color. In this talk we discuss examples and problems that help students acquire proper intuition regarding the actually infinite as part of an introductory Real Analysis course. (Received September 21, 2011)