

1077-42-1083

Nicholas Boros* (borosnic@msu.edu), Department of Mathematics, Michigan State University, East Lansing, MI 48823. *The L^p -Operator Norm of a Perturbation of the Martingale Transform.*

Given a sequence of martingale differences, Burkholder found the sharp constant for the L^p -norm of the corresponding martingale transform. We are able to determine the sharp L^p -norm of small “quadratic perturbations” of the martingale transform in L^p . By “quadratic perturbation” of the martingale transform we mean the L^p norm of the square root of the squares of the martingale transform and the original martingale (with small constant). The problem of perturbation of martingale transform appears naturally if one wants to estimate the linear combination of Riesz transforms (as, for example, in the case of Ahlfors–Beurling operator). Let $\{d_k\}_{k \geq 0}$ be a complex martingale difference in $L^p[0, 1]$, where $1 < p < \infty$, and $\varepsilon_k \in \{\pm 1\}, \forall k$. If $\tau^2 \leq p^* - 1$ and $n \in \mathbb{Z}_+$ then

$$\left\| \sum_{k=0}^n \begin{pmatrix} \varepsilon_k \\ \tau \end{pmatrix} d_k \right\|_{L^p([0,1], \mathbb{C}^2)} \leq ((p^* - 1)^2 + \tau^2)^{\frac{1}{2}} \left\| \sum_{k=0}^n d_k \right\|_{L^p([0,1], \mathbb{C})},$$

where $((p^* - 1)^2 + \tau^2)^{\frac{1}{2}}$ is sharp and $p^* - 1 = \max\{p - 1, \frac{1}{p-1}\}$. For $2 \leq p < \infty$ the result is also true with sharp constant for $\tau \in \mathbb{R}$. (Received September 16, 2011)