1077-14-28 **Allen Knutson*** (allenk@math.cornell.edu), NY. *A stratification of the space of all $k$-planes in $\mathbb{C}^n$.*

The space of all $k$-dimensional linear subspaces of $\mathbb{C}^n$ forms a manifold called the **Grassmannian**. To describe a $V$ in it, pick a basis, and put the resulting $k \times n$ matrix in reduced row-echelon form. Based on the $k$ locations of the pivot columns, we can break the Grassmannian into $\binom{n}{k}$ pieces. But we actually want to refine that:

1. Consider the $k \times n$ *real* matrices, such that every $k \times k$ determinant, or **Plücker coordinate**, is nonnegative. Lusztig defined a stratification in which the boundary of each stratum is a union of others.

2. There is a natural deformation of the Plücker coordinate ring to a noncommutative one. Very few ideals survive this; the column-scaling-invariant ones that do define a list of closed subsets.

3. If we use subspaces of $(\mathbb{F}_p)^n$ instead, the map $r \mapsto r^p$ on the Plücker ring enjoys $(a+b)^p = a^p + b^p$ (the **Freshman's Dream**). One can define a good "$p$th root" map $\phi$, and ask which ideals $I$ are preserved by $\phi$.

Amazingly, these three very different sources all give the same stratification. I will describe the strata, and how to naturally index them by juggling patterns of length $n$ with $k$ balls. (Received September 19, 2011)