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Hafedh Herichi and **Michel L. Lapidus*** (lapidus@math.ucr.edu). *Fractal Strings, the Riemann Hypothesis, Universality and Phase Transitions.*

In the first part of our joint memoir in preparation with Hafedh Herichi, we provide a precise functional analytic framework for studying the spectral operator $a = a_c$, acting on the class of generalized fractal strings of a given dimension c , as introduced semi-heuristically by M. van Frankenhuysen and the presenter in their 2006 Springer research monograph. After having defined the spectral operator a (and its T -truncations $a - T$) as a suitable meromorphic function of the infinitesimal shift of the real line, we determine its spectrum (and that of $a - T$). We deduce that the Riemann hypothesis is true if and only if the spectral operator a_c is quasi-invertible (i.e., each truncation $a - T$ is invertible) for every c other than $1/2$. Using results concerning the universality of the Riemann zeta function, we also show that the spectral operator is invertible for $c > 1$, not invertible for $1/2 < c < 1$, and conditionally (i.e., under the Riemann hypothesis), is invertible for $0 < c < 1/2$. Furthermore the spectrum of a_c is bounded for $c > 1$, the entire complex plane \mathbf{C} for $1/2 < c < 1$, and unbounded but conditionally, not all of \mathbf{C} , for $0 < c < 1/2$. We therefore establish that several types of phase transitions occur at $c = 1/2$ and at $c = 1$. (Received September 20, 2011)