1024-20-78 **Michael D. Fried*** (`mfried@math.uci.edu`), 3547 Prestwick Rd, Billings, MT 59101. *Maximal Frattini quotients of p-Poincaré Mapping class groups.*

The Main Conjecture on Modular Towers (**MT**s) relates the R(egular)I(nverse)G(alois)P(roblem) and the S(trong)T(orsion)C(onjecture).

Assume G is p-perfect (no $\mathbb{Z}/p$ quotient, but p divides $|G|$). Proving the Main Conjecture uses group extensions $M_{G,O}$ of G by the p-completion of a fundamental group: O is a braid orbit on a set defined by p' conjugacy classes $\mathrm{C}_1, \dots, \mathrm{C}_r = \mathbf{C}$ in G.

Three [F(rattini)P(rinciple)s] combinatorially interpret geometric cusps on tower levels attached to $(G, \mathbf{C}, p)$. When $r = 4$, levels are upper half-plane quotients covering the j-line. A cusp is a p *cusp* if p divides its ramification index.

[FP1] interprets p cusps combinatorially.

[FP2] is a condition guaranteeing an infinite sequence of cusps.

[FP3] is an iff condition for all cusps over a given one to be p-cusps.

These and the *Fried-Serre Spin Lifting formula* show how to produce p-cusps when $p = 2$ (so proving the Main Conjecture).

We use the graphical shift-incidence matrix coming from a pairing on cusps. Allows comparing general **MT** cusps with those on modular curves. (Received December 29, 2006)