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David J Saltman* (saltman@math.utexas.edu), Department of Mathematics, University of Texas at Austin, 1 University Station C1200, Austin, TX 78712. *Division algebras and regular local rings*. Preliminary report.

Perhaps the most important open question in the theory of division algebras is one of the oldest. Namely, is every division algebra of prime degree cyclic? This can be rephrased as follows. Let D/F be a division algebra of prime degree q (F is the center of D). Is there a degree q Galois extension L/F such that L splits D ? That is, such that $D \otimes_F L$ is the matrix algebra $M_q(L)$? Of course, the isomorphism classes of division algebras D'/F form the Brauer group $\text{Br}(F)$, and “split” means equal to the identity in the Brauer group. Often, one can attack splitting questions through ramification. Recall that any discrete valuation domain $R \subset F$ with field of fractions $q(R) = F$ defines a ramification map $\text{Br}(F) \rightarrow H^1(\bar{R}, \mathbb{Q}/\mathbb{Z})$. Thus a partial answer to our big question would be provided if we could prove the conjecture that for every D/F of degree q , there is a degree q Galois L/F such that L splits all the ramification of D , where “all” is suitably defined. We will talk about our work on this conjecture, and in particular about understanding $\text{Br}(F)/\text{Br}(S)$, where S is a regular local ring with $q(S) = F$. (Received January 06, 2007)