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Maosheng Xiong* (xiong@math.uiuc.edu), 273 Altgeld Hall, MC-382, Dept. of Math, Univ. of Illinois at UC, 1409 W. Green Street, Urbana, IL 61801, and **Alexandru Zaharescu** (zaharesc@math.uiuc.edu), Dept. of Math, Univ. of Illinois at UC, 1409 W. Green Street, Urbana, IL 61801. *Selmer groups and Tate-Shafarevich groups for the congruent number problem.*

We study the distribution of the sizes of the Selmer groups arising from the three different 2-isogenies and their dual 2-isogenies for the elliptic curve $E_n : y^2 = x^3 - n^2x$. We show that three of them are almost always trivial, while the 2-rank of the other three follows a Gaussian distribution. It implies three almost always trivial Tate-Shafarevich groups and three large Tate-Shafarevich groups. When combined with a result obtained by Heath-Brown, we show that the mean value of the 2-rank of the large Tate-Shafarevich groups for square-free positive odd integers $n \leq X$ is $\frac{1}{2} \log \log X + O(1)$ as $X \rightarrow \infty$. (Received November 09, 2006)