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TX 77845. Causality and Frame Theory. Preliminary report.

The notion of causality leads to a natural equivalence relation on frame sequences that seems to be new in the literature. The relation that is established between a Riesz basis and the orthonormal basis derived from it by the Gram-Schmidt procedure is causal in the sense that the k'th element of the orthonormal basis is a linear combination of only the first k elements of the Riesz basis. It is not hard to show that every frame sequence is causally related in this sense to a Parseval frame, and if the frame is not a Riesz basis there are uncountably many such Parseval frames. Causal relationship under very mild restriction is an equivalence relation. The fact that there are many causually equivalent yet non-unitarily equivalent Parseval frames shows that the theory is rich. For a given frame we consider best Parseval approximants within its causal equivalence class. This yields more than one natural causal Gram-Schmidt procedure for coverting a frame to a Parseval frame. (Received September 19, 2005)