We prove that a set of n unit balls in $R^{d}$ admits at most four distinct geometric permutations when $n$ is sufficiently large, thus settling a long-standing conjecture in combinatorial geometry. Our results were subsequently improved by Cheong, Goaoc and Na to two if $n \geq 9$ and three otherwise.

The constant bound for unit balls significantly improves upon the $\Theta\left(n^{d-1}\right)$ bound for balls of arbitrary radii. Intrigued by this large gap between the two bounds, we also prove a bound of $O\left(\gamma^{\log \gamma}\right)$ on the geometric permutations of balls when the radius ratio between the largest to smallest balls is bounded by $\gamma$, and a tight bound of $2^{d-1}$ on the geometric permutations of $n$ disjoint rectangular boxes in $R^{d}$. (Received August 09, 2005)

