1010-47-106 **Quoc-Phong Vu*** (qvu@math.ohiou.edu), Department of Mathematics, Morton Hall 561, Athens, OH 45701. *A new proof and generalizations of Gearhart's theorem.*

The well known Gearhart's Theorem states that if $T(t), t \geq 0$, is a strongly continuous semigroup with generator $A$, on a Hilbert space, then the following are equivalent: (i) For every $\omega$-periodic (continuous) function $f$ (with values in $H$), the equation $u'(t) = Au(t) + f(t)$ has a unique $\omega$-periodic (continuous) solution; (ii) The set $\{\frac{i2\pi k}{\omega} : k = 0 \pm 1, \pm 2, ...\}$ is contained in the resolvent set of $A$ and the resolvent operators $\|(\frac{i2\pi k}{\omega} - A)^{-1}\|$ are uniformly bounded (on this set); (iii) $1 \notin \sigma(T(\omega))$.

We present a new proof and generalizations of the Gearhart's theorem. The new approach is based, from one side, on the theory of almost periodic functions with values in Hilbert space, in particular on Parseval's equality for almost periodic functions, and from the other side, on results on sums of commuting operators (which are closely related to results on Lyapunov-Sylvester operator equations). The generalizations are into two main directions; namely, (i) to more general classes of equations than the equation $u'(t) = Au(t) + f(t)$ and (ii) to more general classes of functions than periodic functions. (Received August 22, 2005)