Meeting: 1006, Lubbock, Texas, SS 2A, Special Session on Differential Geometry and Its Applications

1006-53-212 **Katrin Leschke*** (leschke@math.umass.edu), Department of Mathematics and Statistics, Lederle Graduate Research Tower, University of Massachusetts, Amherst, MA 01003. *Willmore tori with non-trivial normal bundle.*

A classical result states that a harmonic map $f : M \to S^2$ from a compact Riemann surface M of degree $|deg f| > g - 1$ is holomorphic or antiholomorphic where g is the genus of M. In view of the close resemblance between harmonic maps into S^2 and Willmore surfaces in S^4, we expect a similar criterion to hold for Willmore surfaces. In case of genus 0, Bryant and subsequently Ejiri and Montiel show that all Willmore spheres are given by complex holomorphic data. The case of higher genus Willmore surfaces is more involved.

A Willmore surface $f : M \to S^4$ of a compact Riemann surface into the 4-sphere gives a new branched conformal immersion by using the $(1, 0)$–part of the derivative of the conformal Gauss map. This map, the so–called Bäcklund transform of f, is again Willmore. Using the sequence of Bäcklund transforms, we obtain a generalization of Bryant's result: If $f : T^2 \to S^4$ is a Willmore torus in S^4 with non–trivial normal bundle then f comes from a twistor projection of an elliptic curve in $\mathbb{CP}^3$ or from a minimal torus with planar ends in $\mathbb{R}^4$. (Received February 15, 2005)