Meeting: 1006, Lubbock, Texas, SS 12A, Special Session on Graph Theory

1006-05-36 Heather J Gavlas and Joy M Morris* (joy.morris@uleth.ca), Dept of Math & CS, University of Lethbridge, Lethbridge, Alberta T1K 6R4, Canada. Cyclic Hamiltonian (Near-)Decompositions of the Complete Graph.

It has been proven that the complete graph on n vertices can be decomposed into hamiltonian cycles, whenever n is odd. Similarly, if we remove a 1-factor (perfect matching) from the complete graph on an even number of vertices, the remaining graph can always be decomposed into hamiltonian cycles; this is what is referred to as a near-decomposition.

To make the problem interesting again, we put constraints on the hamiltonian cycles that we allow to be in the decomposition. A cyclic hamiltonian decomposition of the complete graph is a decomposition of the complete graph into hamiltonian cycles, in such a way as to ensure that rotating any cycle in the decomposition gives us a (possibly different) cycle in the decomposition.

It has been proven that a cyclic hamiltonian decomposition of the complete graph on n vertices always exists when n is odd, as long as $n \neq 15$ and $n \neq p^{\alpha}$, where p is prime and $\alpha > 1$, and that these constraints are necessary. We prove that when n is even, cyclic hamiltonian near-decompositions of the complete graph on n vertices exist if and only if $n \neq 2p^{\alpha}$ where p is prime, and n is either 2 or 4 (mod 8). (Received January 18, 2005)