Meeting: 1001, Evanston, Illinois, SS 7A, Special Session on Geometric Partial Differential Equations

1001-76-120 William P. Ziemer* (ziemer@indiana.edu), Mathematics Dept, Indiana Univesity, Bloomington, IN 47495. The normal trace of vector fields with weak divergences. Preliminary report.

We will discuss some current work and some recent progress that have been made in an area that was initiated by Gui-Qiang Chen, namely, the normal behavior of vector fields $F \in L^p(\mathbb{R}^n; \mathbb{R}^n)$, $1 \le p \le \infty$, whose divergences are Radon measures. A result that is critical to our investigations is the following

Theorem (Fuglede) Suppose $F \in L^p(\mathbb{R}^n; \mathbb{R}^n)$, $1 \le p \le \infty$, is a vector field with divF = f, where $f \in L^1$. Then there exists a function $g: \mathbb{R}^n \to \mathbb{R}$ with $g \in L^p$ such that

$$\int_E divF = \int_E f = \int_{\partial^* E} F(y) \cdot \nu(y) \, dH^{n-1}(y)$$

for all sets of finite perimeter E except possibly those for which

$$\int_{\partial^* E} g \, dH^{n-1} = \infty$$

This was recently used to establish the following:

Theorem With F as above and with $divF = \mu$, where μ is a signed measure, let E be an open set with Lipschitz boundary. Then, M. Torres recently proved that there exists a measure σ on ∂E such that

$$\mu(E) := \int_E divF = \int_{\partial E} \sigma.$$

If $p = \infty$, the result remains true for sets E with finite perimeter. We will discuss the possibility of this result remaining true for $1 \le p \le \infty$. (Received August 18, 2004)