Meeting: 1001, Evanston, Illinois, SS 16A, Special Session on Spectral Problems of Differential Operators

1001-47-26 Plamen Djakov* (djakov@fmi.uni-sofia.bg), Department of Mathematics, Sofia University, Bulv. J. Bourchier 5, 1164 Sofia, Bulgaria, and Boris Mityagin, The Ohio State University. Asymptotics of spectral gaps of 1D periodic Schroedinger operators with two term potentials.

Formulas for the asymptotics of spectral gaps γ_n of 1D periodic Schrödinger operator $L = d^2/dx^2 + v(x)$, $x \in \mathbb{R}$, are obtained for two term potentials $v(x) = a \cos 2x + b \cos 4x$, where a, b are real, and $a, b \neq 0$.

Let us write a and b in the form $a = -4\alpha t$, $b = -2\alpha^2$, where α, t are both real if b < 0, and both pure imaginary if b > 0. Then, for fixed t, n and small enough α

$$\gamma_n = \frac{\pm 8\alpha^n}{2^n [(n-1)!]^2} \prod_{k=1}^{n/2} \left(t^2 - (2k-1)^2 \right) \left((1+O(\alpha)) \quad \text{for even } n, \right)$$

$$\gamma_n = \frac{\pm 8\alpha^n t}{2^n [(n-1)!]^2} \prod_{k=1}^{(n-1)!} \left(t^2 - (2k)^2\right) \left((1+O(\alpha)) \text{ for odd } n.$$

If α and t are fixed, then for large enough n

$$\gamma_n = \frac{8|\alpha/2|^n}{[2 \cdot 4 \cdots (n-2)]^2} \left| \cos\left(\frac{\pi}{2}t\right) \right| \left[1 + O((\log n)/n) \right] \text{ for even } n,$$

$$\gamma_n = \frac{8|\alpha/2|^n}{[2 \cdot 4 \cdots (n-2)]^2} \frac{2}{\pi} \left| \sin\left(\frac{\pi}{2}t\right) \right| \left[1 + O((\log n)/n) \right] \text{ for odd } n.$$

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