Meeting: 1001, Evanston, Illinois, SS 22A, Special Session on Special Functions, Orthogonal Polynomials, and their Applications

1001-33-96 Willard Miller* (miller@ima.umn.edu), School of Mathematics, University of Minnesota, 127 Vincent Hall, 206 Church St. SE, Minneapolis, MN 55455. Second-order superintegrable systems on conformally flat spaces.

We say that a Schrödinger operator with potential on a Riemannian space is 2nd-order superintegrable if there are 2n - 1 (classically) functionally independent 2nd order symmetry operators. (The 2n - 1 is the maximum possible number of such symmetries.) These systems are of considerable interest in special function theory because they are multiseparable, i.e., variables separate in several coordinate systems, and are explicitly solvable in terms of special functions.

First we give examples of quantum superintegrable systems. The energy observable is degenerate and the 2nd order symmetries close quadratically under repeated commutation. Virtually all of the special functions of mathematical physics (in one and several variables) arise in this study and formulas expanding one type of special function as a series or integral in another type emerge as a byproduct.

Finally, we present very recent results giving the general structure of superintegrable systems in all 2D, and 3D conformally flat spaces, and a complete list of such spaces and potentials in 2D. The results reported here were obtained in collaboration with E.G. Kalnins, G.S.Pogosyan and J. Kress. (Received August 16, 2004)