Meeting: 1001, Evanston, Illinois, SS 22A, Special Session on Special Functions, Orthogonal Polynomials, and their Applications

1001-33-316 **Ahmed I. Zayed*** (azayed@math.depaul.edu), Department of Mathematical Sciences, DePaul University, 2310 N. Kenmore Ave., Chicago, IL 60614. *A Generalization of the Prolate Spheroidal Wave Functions.* Preliminary report.

The prolate spheroidal wave functions (PSWF) are eigenfunctions of the boundary-value problem:

$$\frac{d}{dz}\left((1-z^2)\frac{dw}{dz}\right) + c^2(1-z^2)w = -\lambda w, \quad -1 \leq z \leq 1,$$

with $w(\pm 1)$ being bounded, where $c > 0$. They reduce to the Legendre polynomials when $c = 0$. The PSWF seem to possess unique properties that no other known systems possess, such as being orthogonal in $L^2(-1,1)$ and $L^2(-\infty,\infty)$. When appropriately normalized, they satisfy the relations:

$$\int_{-1}^{1} \phi_m(x)\phi_n(x)dx = d_n\delta_{m,n}, \int_{-\infty}^{\infty} \phi_m(x)\phi_n(x)dx = \delta_{m,n},$$

for some $d_n > 0$. Moreover, they form an orthogonal basis for $L^2(-1,1)$ and for a subspace of $L^2(-\infty,\infty)$, known as the Paley-Wiener space PW_c, which consists of all entire functions $f(z)$ satisfying

$$|f(z)| \leq \sup_x |f(x)| \exp\left(c|y|\right), \quad \int_{-\infty}^{\infty} |f(x)|^2\, dx < \infty$$

In this talk we show that the prolate spheroidal wave functions are not really unique in that sense. We shall show how to construct other systems with similar properties. (Received August 30, 2004)