

Meeting: 1001, Evanston, Illinois, SS 22A, Special Session on Special Functions, Orthogonal Polynomials, and their Applications

1001-33-227 **Paul M Terwilliger*** (terwilli@math.wisc.edu), Math Department, University of Wisconsin, 480 Lincoln Drive, Madison, WI 53706. *The compact representation of a Leonard pair*. Preliminary report.

Let K denote a field, and let V denote a vector space over K with finite positive dimension. We consider a pair of linear transformations $A : V \rightarrow V$ and $A^* : V \rightarrow V$ that satisfy both conditions below: (i) There exists a basis for V with respect to which the matrix representing A is irreducible tridiagonal and the matrix representing A^* is diagonal; (ii) There exists a basis for V with respect to which the matrix representing A^* is irreducible tridiagonal and the matrix representing A is diagonal. We call such a pair a *Leonard pair* on V . There is a natural correspondence between the Leonard pairs and a class of orthogonal polynomials consisting of the q -Racah and related polynomials in the Askey Scheme. Let A, A^* denote a Leonard pair on V . Associated with this pair is a certain parameter q that is used to describe the eigenvalues. For the case $q \neq 1, q \neq -1$, we display a basis for V with respect to which the matrix representing $AA^* - qA^*A$ is upper triangular and the matrix representing $A^*A - qAA^*$ is lower triangular. With respect to this basis the matrices representing A, A^* are tridiagonal, with entries of an attractive form. We call this basis the *compact basis*. This is joint work with Hjalmar Rosengren. (Received August 27, 2004)