Meeting: 999, Nashville, Tennessee, SS 2A, Special Session on Wavelets, Frames, and Sampling

999-42-16 **Radu Balan** (rvbalan@scr.siemens.com), Siemens Research Corporation, 755 College Road East, Princeton, NJ 08540, **Peter G. Casazza** (pete@math.missouri.edu), Department of Mathematics, University of Missouri, Columbia, MO 65211, **Christopher Heil*** (heil@math.gatech.edu), School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332, and **Zeph Landau** (landau@sci.ccny.cuny.edu), Department of Mathematics, City College of New York, New York, NY 10031. *Density, Overcompleteness, and Localization of Frames.*

We present a quantitative framework for describing the overcompleteness of a large class of frames. We introduce notions of localization and approximation between two frames $\mathcal{F} = \{f_i\}_{i\in I}$ and $\mathcal{E} = \{e_j\}_{j\in \mathbf{Z}^d}$, relating the decay of the expansion of the elements of $\mathcal{F}$ in terms of the elements of $\mathcal{E}$. A fundamental set of equalities are shown between two seemingly unrelated quantities: the relative measure, which is determined by certain averages of $\langle f_i, \tilde{f}_i\rangle$ and $\langle e_j, \tilde{e}_j\rangle$ (inner products of frame elements with their corresponding canonical dual frame elements), and the density of the index set I. The above equalities lead to an array of new results that hold for general localized frames. When applied to irregular Gabor frames these recover the Nyquist density theorems as well as giving new results, including relations between frame bounds and density, results on excess, and other quantities. More generally, these apply not only to Gabor frames, where the frame elements are simple time-frequency shifts of a given atom but also to more general systems whose elements share only general envelope of localization in the time-frequency plane. (Received June 16, 2004)