Meeting: 1002, Pittsburgh, Pennsylvania, SS 7A, Special Session on Knots and Macromolecules

1002-52-228 Rob Kusner* (kusner@math.umass.edu), GANG Lab, Department of Mathematics, University of Massachusetts, Amherst, MA 01003, and Wöden Kusner (wkusner@haverford.edu), Haverford College, Haverford, PA 19041. On pearl-number and ropelength of knots and links. Preliminary report.
A pearl necklace is a cyclically ordered collection of points in $R^{3}$ where distances between distinct points are at least 1 and where adjacent points are at distance exactly 1; in other words, the points are centers of disjoint unit (diameter) balls where adjacent balls are in contact, just as in a physical necklace of pearls. The core polygon of a pearl necklace is defined by edges joining adjacent pearl centers. The pearl-number of a knot (or link) is the smallest number of points needed to make pearl necklaces whose core polygons represent the link components. We show the pearl-number of a nontrivial knot or link satisfies $N \geq 12$. In fact, we get an explicit comparison between pearl-number and (minimum) ropelength: if a knot can be made with ropelength $L$, then it can with pearl-number $N<L+1$; and if a knot can be realized with pearl-number $N$, then it can with ropelength $L<C N$, where $C$ is (very close to) $\sqrt{2}$. The pearl number bound for knots then follows from the best known lower bound on the ropelength, though other ways to get $N \geq 12$ will be sketched. Unfortunately, this is not sharp, since the conjectured pearl number of a knot satisfies $N \geq 15$. We also give an explicit example of a trefoil knot with $\mathrm{N}=15$. (Received September 14, 2004)

