Meeting: 1002, Pittsburgh, Pennsylvania, SS 9A, Special Session on Multivariate Hypergeometric Functions: Combinatorial and Algebro-Geometric Aspects

1002-35-131 Askold Khovanskii^{*} (askold@math.toronto.edu), Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3, Canada. Nonsolvability of Holonomic Systems of Equations by Quadratures.

Consider a system of linear differential equations of the following form dy = Ay, where $y = y_l, ..., y_n$ is the unknown vector function and A is an $(n \times n)$ matrix consisting of differential l-forms with rational coefficients in the space C^n , satisfying the condition of complete integrability $dA + A \wedge A = 0$ and having the following form:

$$A = \sum_{i=1}^{k} A_i \frac{dl_i}{l_i},$$

where the A_i are constant matrices and the l_i are linear non-homogeneous functions in C_n . If the matrices A_i can be put at the same time into triangular form, then the system is solvable by quadratures. There undoubtedly exist integrable non-triangular systems. However, when the matrices A_i are sufficiently small such systems do not exist.

Theorem: A completely integrable non-triangular system with the matrices A_i having sufficiently small norm, is strongly nonsolvable by quadratures, i. e., its solution cannot be represented through the germs of all meromorphic functions by means of compositions, arithmetic operations, integrations, differentiations, and solutions of algebraic equations. (Received September 11, 2004)