Meeting: 1002, Pittsburgh, Pennsylvania, SS 8A, Special Session on Graph Polynomials

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Evelin Toumpakari* (evelint@math.uchicago.edu), 5734 S. University Ave, Chicago, IL 60637. *On the sandpile group of regular trees.*

Motivated by statistical physics (self-organized criticality), the sandpile group is an algebraic invariant of graphs. The finite abelian group G=G(X) (the "sandpile group") of a finite rooted graph X with n+1 vertices is defined using the n by n matrix Δ obtained from the Laplacian L=D-A by deleting the row and column corresponding to the root. Here A is the adjacency matrix and D is a diagonal matrix containing the degrees of the vertices. The rows of Δ span a lattice Λ in Z^n. G is the quotient Z^n/Λ. The order of G is the number of spanning trees of X. X is often defined by adding a root ("sink") to a nice graph Y on n vertices in a way that will make all the original vertices have the same degree.

Finding the structure of G seems quite difficult. Dhar et al. found the rank of G for the case when Y is the N by N square grid; but for N by M grids, even finding the rank remains open.

We give detailed structural information about G when Y is a balanced regular tree of degree d and height h. We compute the rank, exponent, order of these groups and state a conjecture on the rank of each Sylow subgroup. We find a cyclic Hall-subgroup of order $(d-1)^h$. We find that the base-$(d-1)$-logarithm of the exponent is asymptotically $3h^2/\pi^2$. (Received July 20, 2004)