TEACHING MATH ONLINE: THEORY INTO PRACTICE

Chad M. Topaz

Institute for the Quantitative Study of Inclusion, Diversity, and Equity, 501(c)(3), and
Williams College, Dept. of Mathematics and Statistics

PRELIMINARIES

WHEN I GIVE RESEARCH TALKS

WHEN I GIVE RESEARCH TALKS

WHEN I GIVE TEACHING TALKS

WHEN I GIVE TEACHING TALKS

WHY AM I HERE?

- ~~To critique your college/university~~

- ~~To critique your department~~

- ~~To find fault with what you do in the classroom~~

- ~~To tell you what values you should have~~

- To share my current thinking

MY PRIORITIES IN LIFE RIGHT NOW

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

- Fighting the spread of fascism

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

- Fighting the spread of fascism

- Advocating for public health

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

- Fighting the spread of fascism

- Advocating for public health

- Supporting my students

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

- Fighting the spread of fascism

- Advocating for public health

- Supporting my students

- Maintaining my family's well-being

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

- Fighting the spread of fascism

- Advocating for public health

- Supporting my students

- Maintaining my family's well-being

- Tending to my mental health

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

- Fighting the spread of fascism

- Advocating for public health

- Supporting my students

- Maintaining my family's well-being

- Tending to my mental health

- Tending to my students' mental health

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

- Fighting the spread of fascism

- Advocating for public health

- Supporting my students

- Maintaining my family's well-being

- Tending to my mental health

- Tending to my students' mental health

- etc.

- etc.

- etc.

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

- Fighting the spread of fascism

- Advocating for public health

- Supporting my students

- Maintaining my family's well-being

- Tending to my mental health

- Tending to my students' mental health

- etc.

- etc.

- etc.

- Season 5 of Queer Eye

MY PRIORITIES IN LIFE RIGHT NOW

- Stopping police killing of Black people

- Fighting the spread of fascism

- Advocating for public health

- Supporting my students

- Maintaining my family's well-being

- Tending to my mental health

- Tending to my students' mental health

- etc.

- etc.

- etc.

- Season 5 of Queer Eye

- Teaching math

DOES IT EXIST?

Awesome Class   Disastrous Class

In-Person Class

Online Class

DOES IT EXIST?

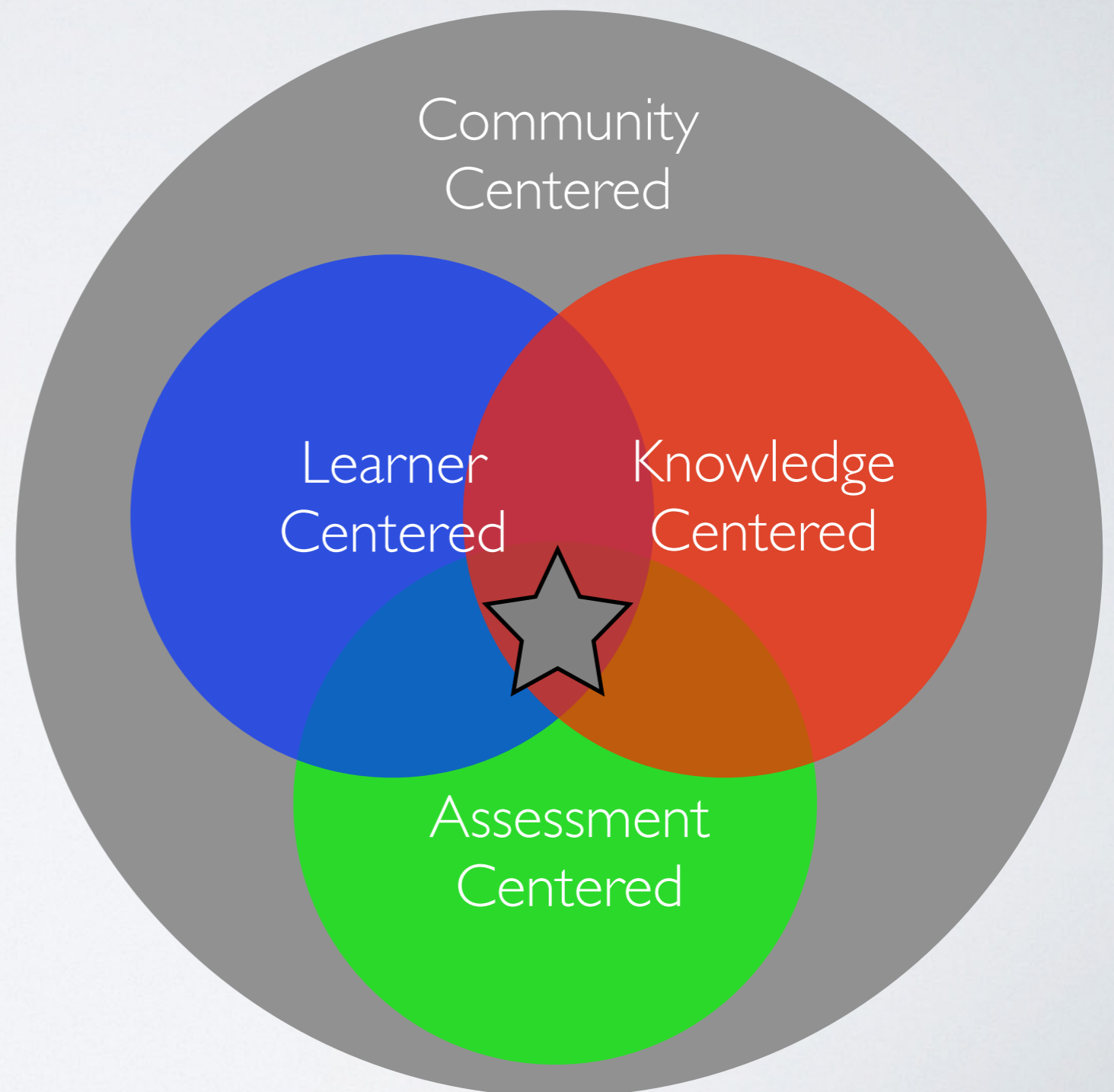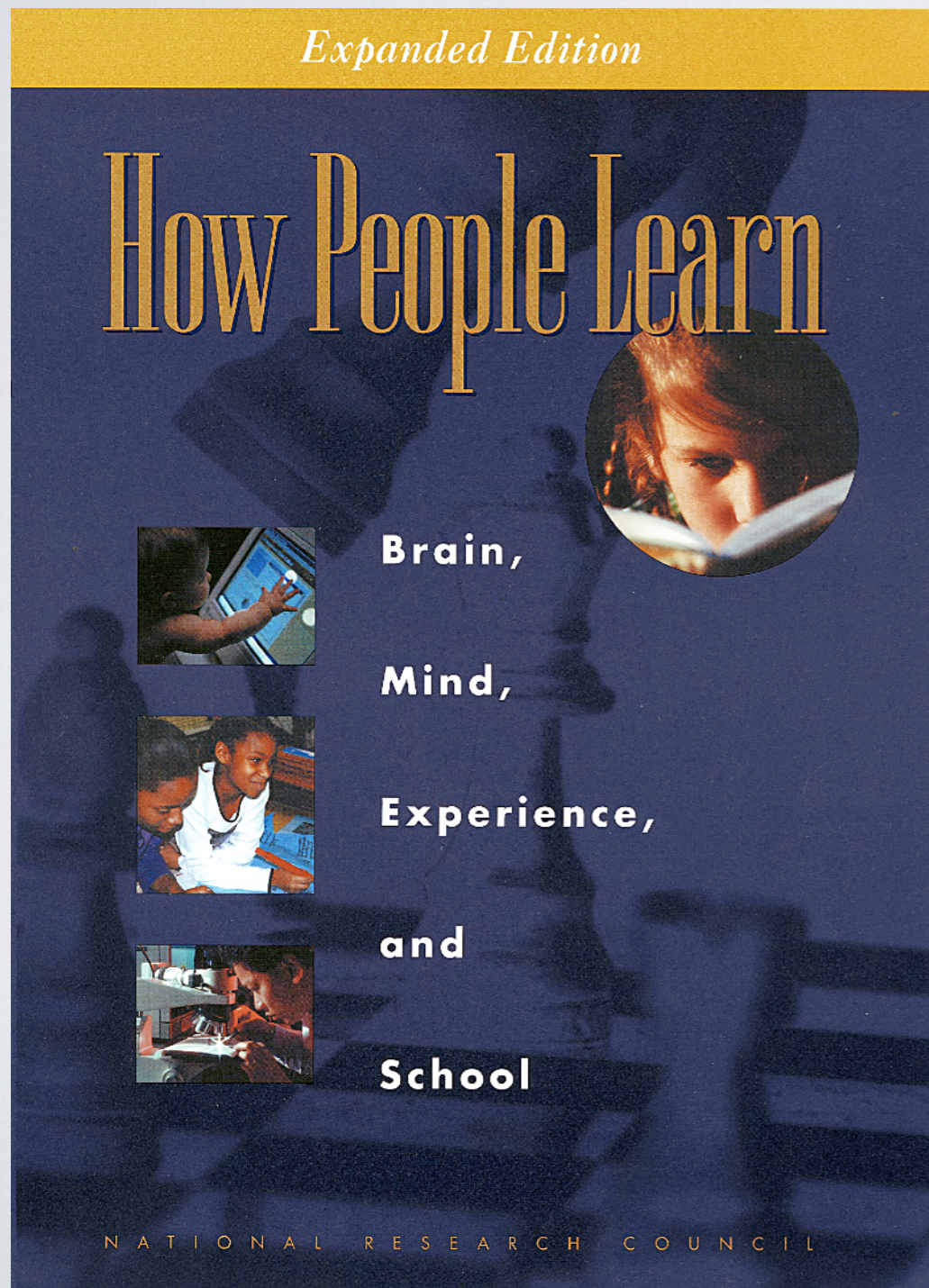|	Awesome Class	Disastrous Class
In-Person Class	✓	✓
Online Class	✓	✓

PLAN FOR THIS TALK

- Ideas from learning science that influence me

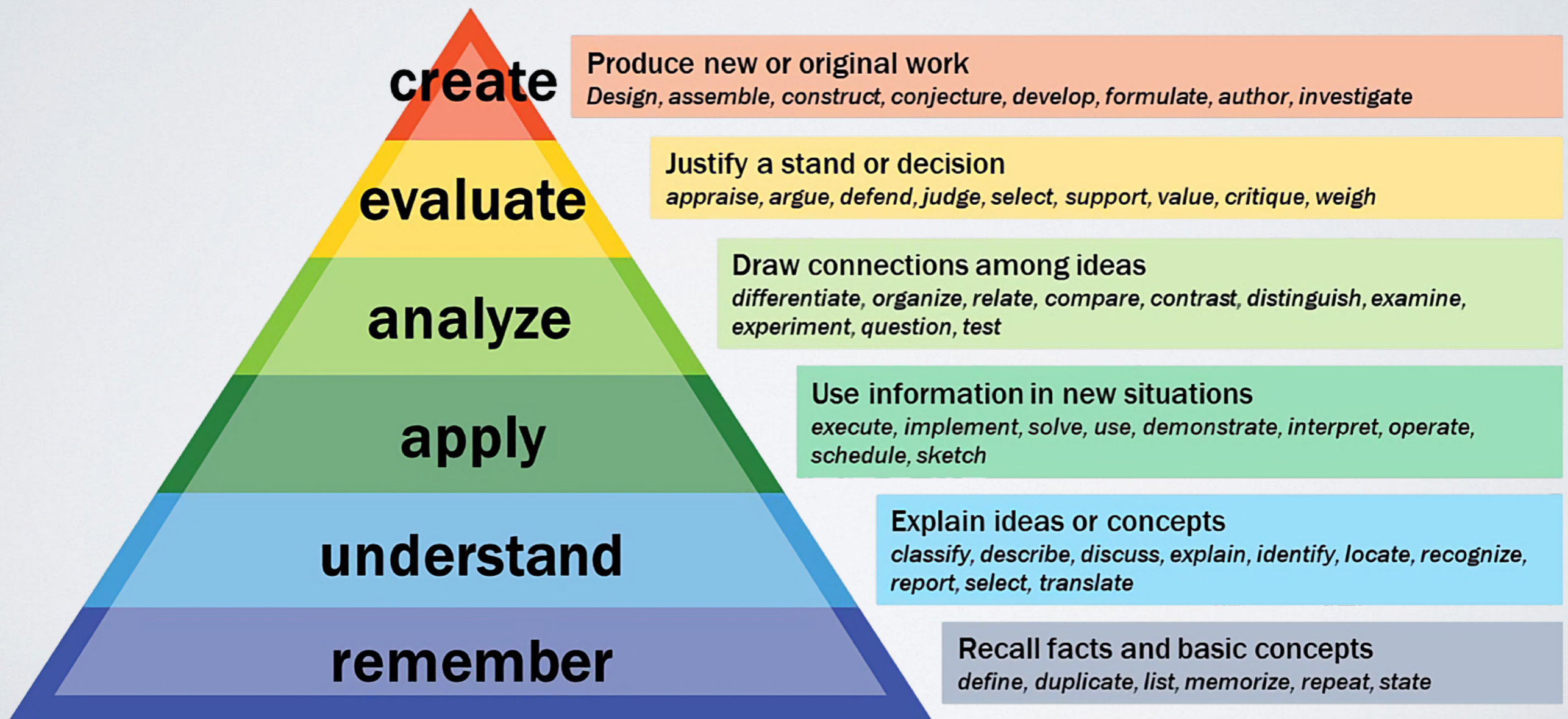- Overview of my online Discrete Math course

A PEDAGOGICAL MODEL
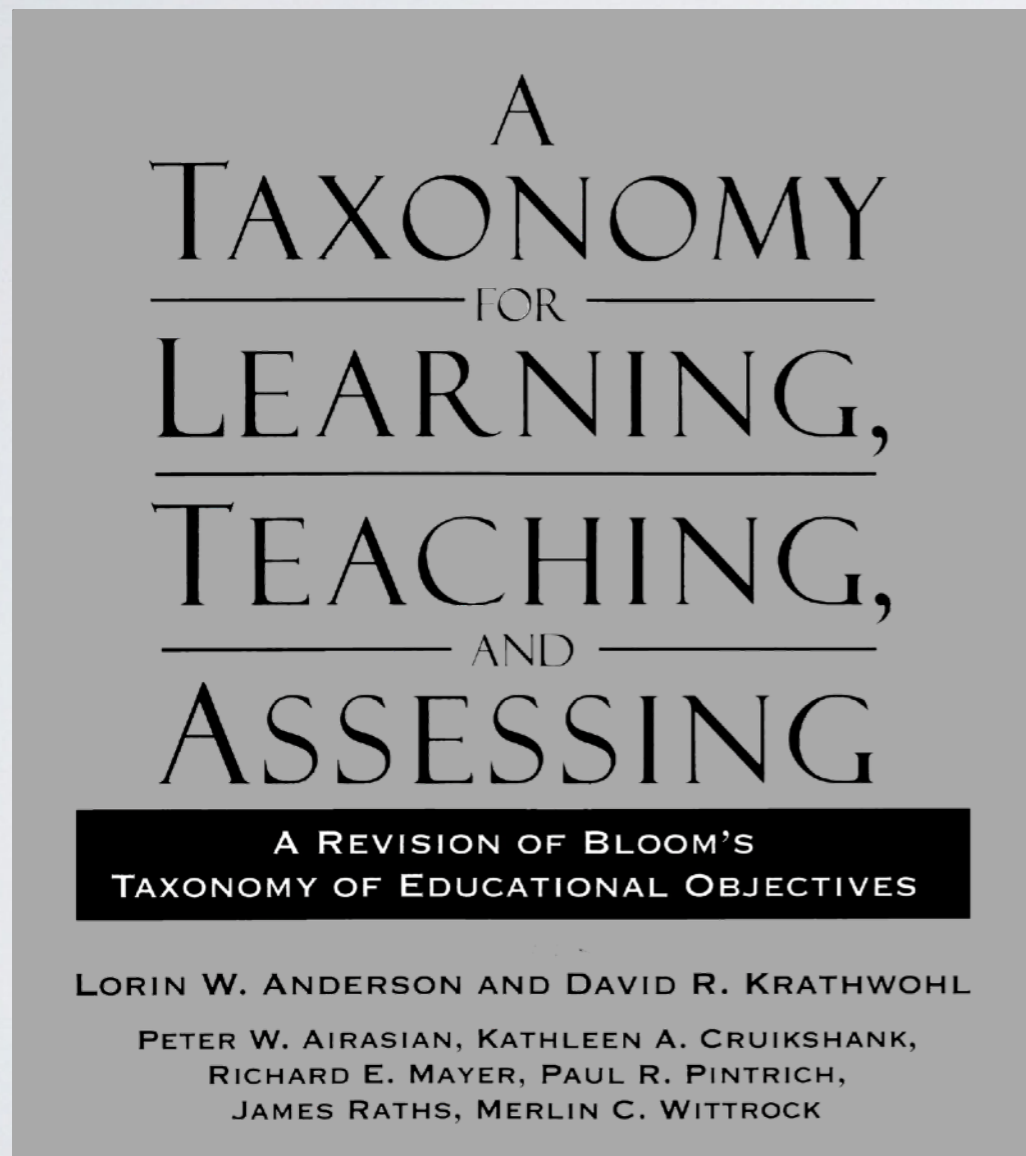
THE DESIGN OF LEARNING ENVIRONMENTS

KNOWLEDGE

A REVISION OF BLOOM'S TAXONOMY

A TAXONOMY FOR LEARNING, TEACHING, AND ASSESSING

A REVISION OF BLOOM'S TAXONOMY OF EDUCATIONAL OBJECTIVES

LORIN W. ANDERSON AND DAVID R. KRATHWOHL

PETER W. AIRASIAN, KATHLEEN A. CRUIKSHANK, RICHARD E. MAYER, PAUL R. PINTRICH, JAMES RATHS, MERLIN C. WITTROCK

Learning objectives: Students will be able to verb noun.

cognitive process

knowledge type

A REVISION OF BLOOM'S TAXONOMY

	Recall Recognize Remember	Interpret Classify Summarize Compare Understand	Execute Implement Apply	Differentiate Organize Attribute Analyze	Check Critique Evaluate	Generate Plan Produce Create
Factual knowledge						
Conceptual knowledge						
Procedural knowledge						
Metacogn. knowledge						

A REVISION OF BLOOM'S TAXONOMY

|	Recall Recognize **Remember**	Interpret Classify Summarize Compare **Understand**	Execute Implement **Apply**	Differentiate Organize Attribute **Analyze**	Check Critique **Evaluate**	Generate Plan Produce **Create**
Factual knowledge						
Conceptual knowledge						
Procedural knowledge			X			
Metacogn. knowledge						

Students can construct the contrapositive of a given conditional statement.

LEARNING AND ENJOYMENT

ANTAGONISM BETWEEN ACHIEVEMENT AND ENJOYMENT IN ATI STUDIES

"There is evidence that negative correlations between student achievement and their enjoyment of instructional methods exist under certain circumstances… students often reported enjoying the method from which they learned the least."

Clark, *Ed. Psych.* (1982)

MEASURING ACTUAL LEARNING VERSUS FEELING OF LEARNING IN RESPONSE TO BEING ACTIVELY ENGAGED IN THE CLASSROOM

Deslauriers *et al.*, PNAS (2019)
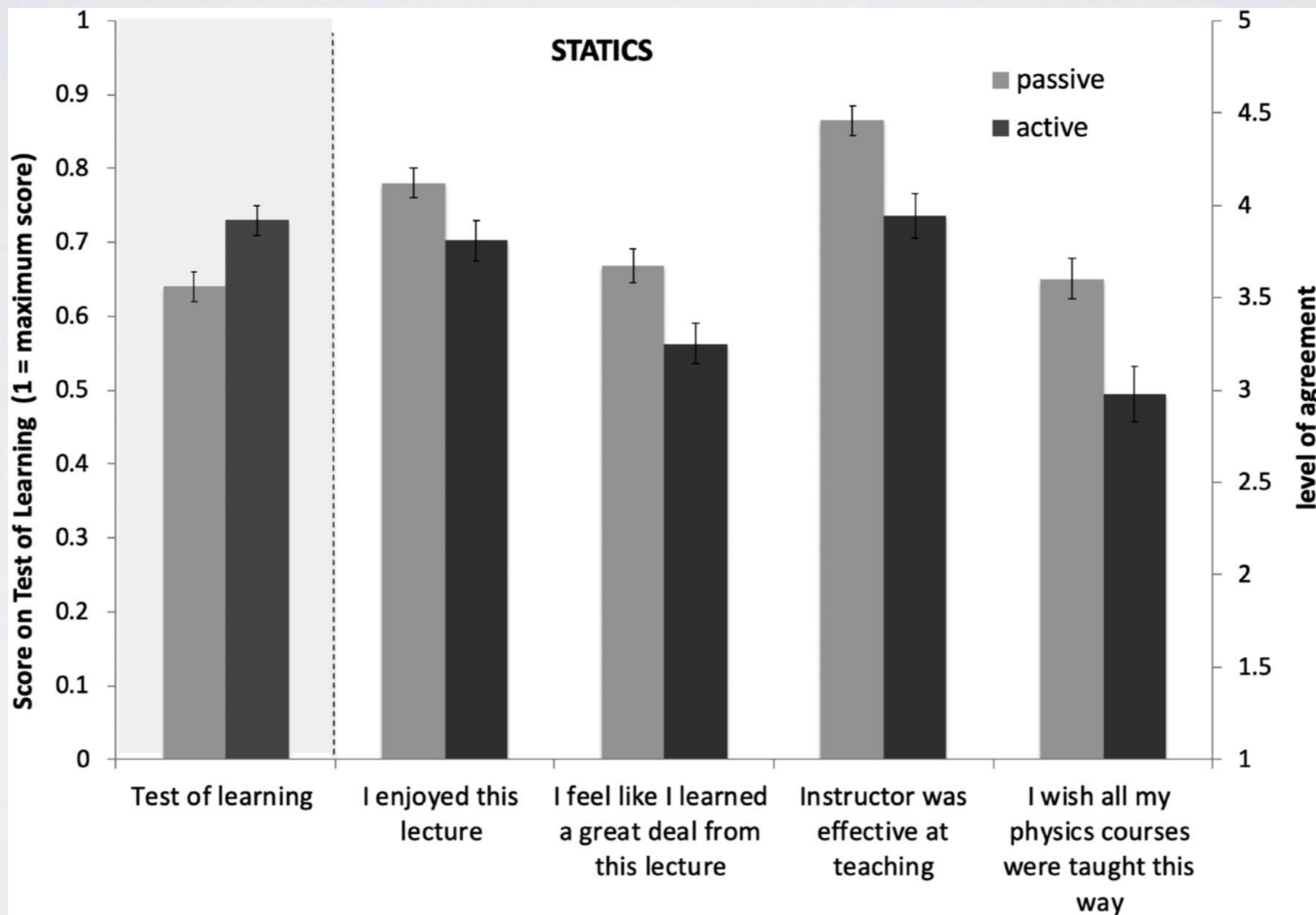
CHAD'S TOTALLY MADE UP MATRIX OF LEARNING AND ENJOYMENT

Effect on student enjoyment

|	−	0	+
+	Avoid	As much as possible	As much as possible
0	Avoid	Fine but useless	Totally fine
−	Avoid	Avoid	Be judicious

Effect on student learning

GOALS AND MOTIVATION

SOME IDEAS ABOUT GOALS AND MOTIVATION

- Posting the grade distribution for this exam will help motivate lower-performing students.

- Contests and competitions will engage and motivate students.

- My students would learn more if they cared less about grades.

- The best students are those who just love math.

- If I show my students the beauty of a topic they'll learn it better.

- If I show my students the value of a mathematical technique they'll learn it better.
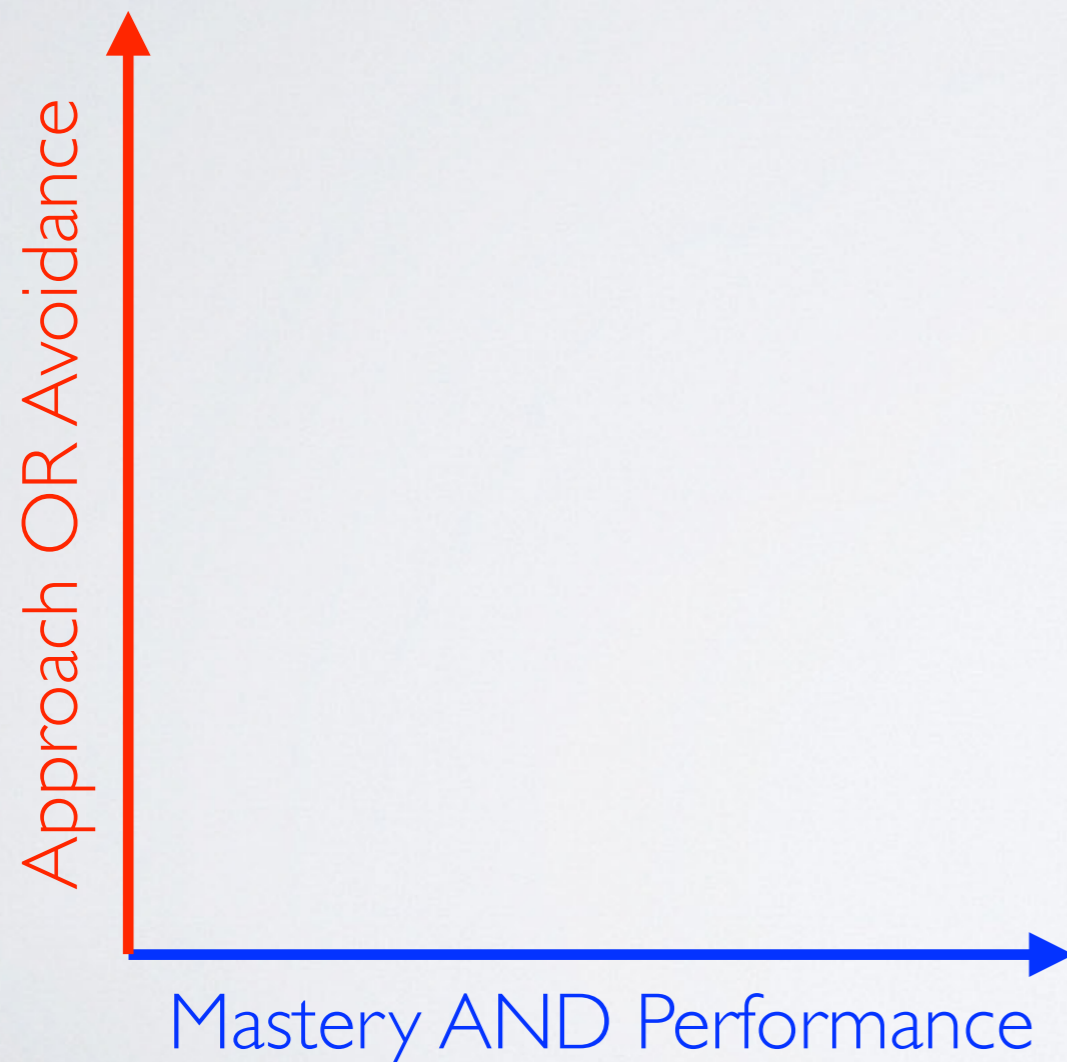
GOAL ORIENTATION

Deshon and Gillespie (2005)
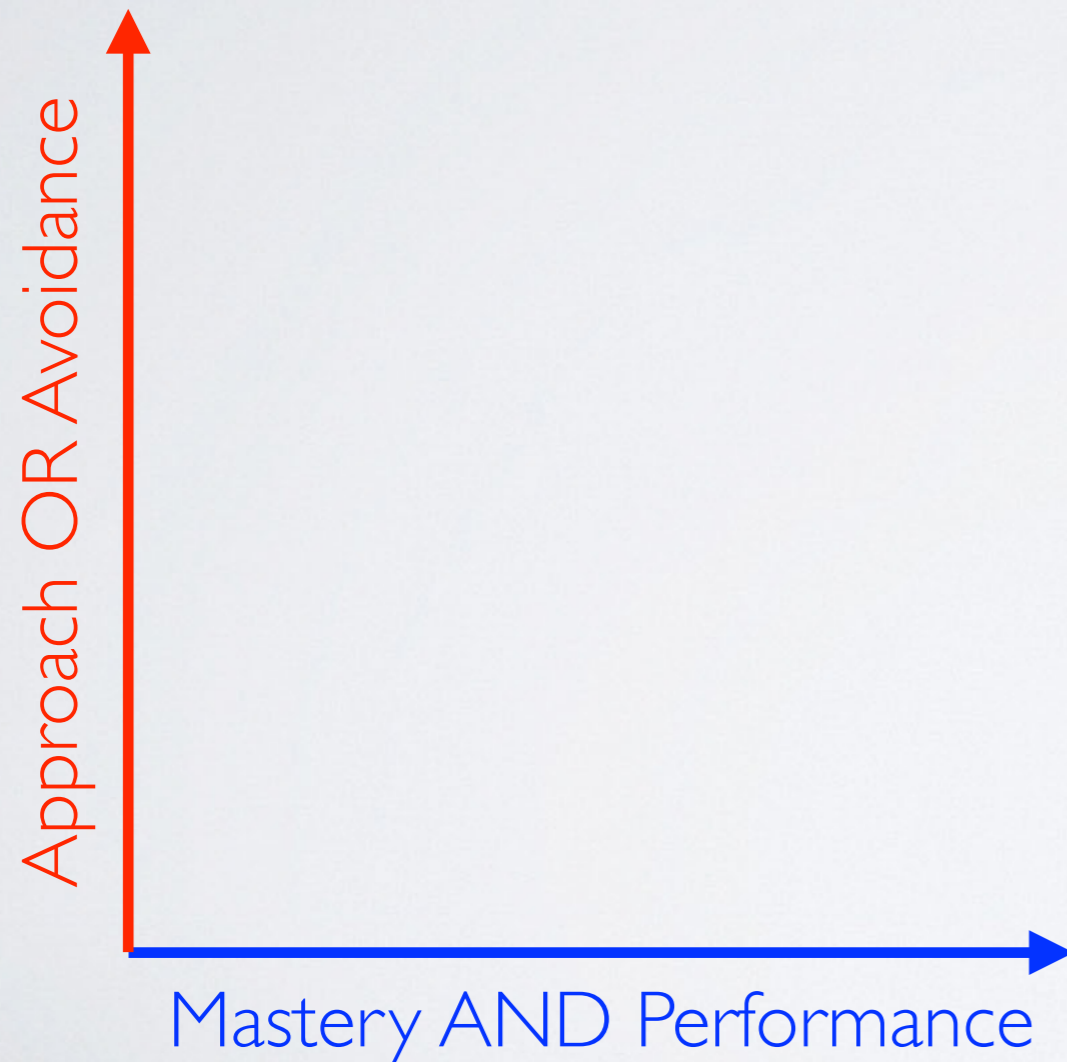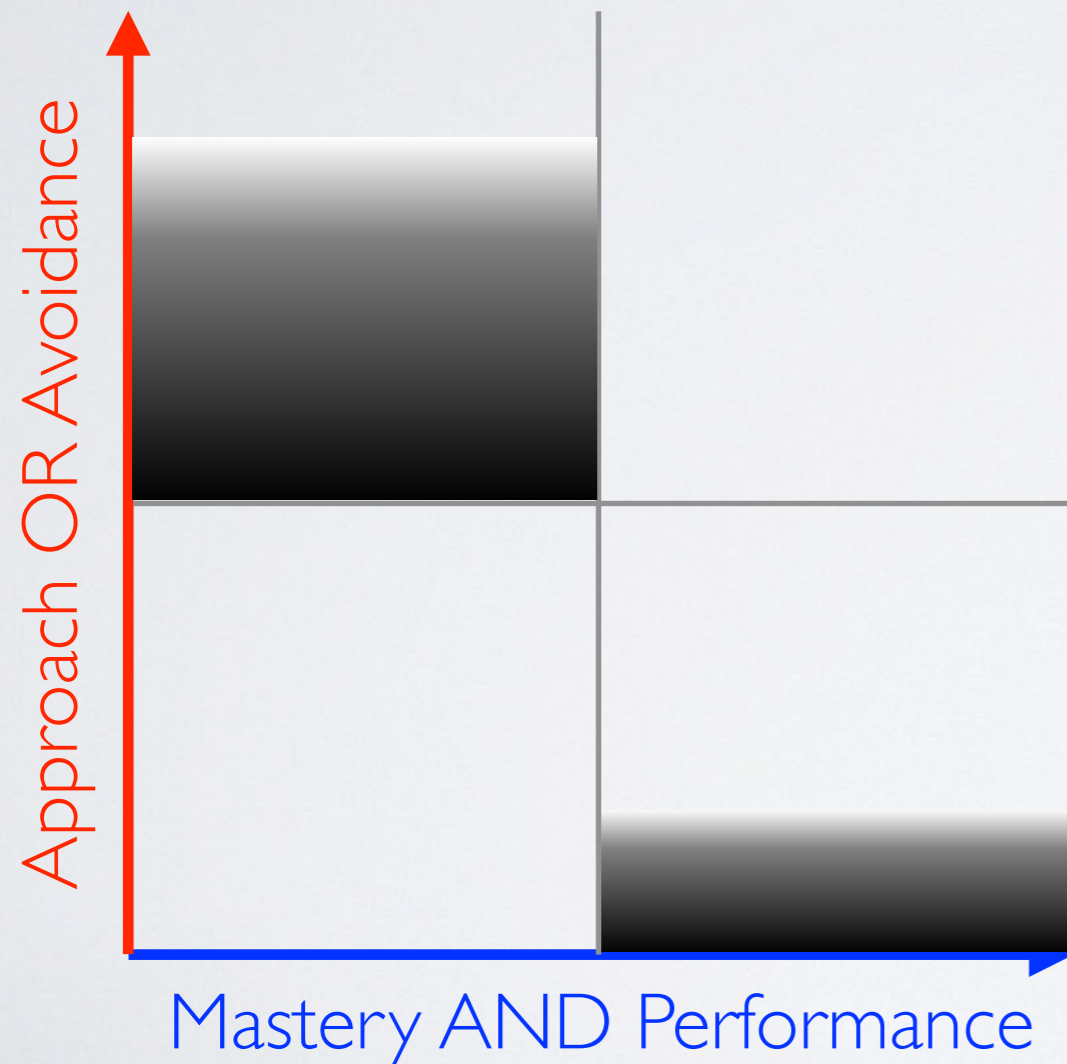
GOAL ORIENTATION

Deshon and Gillespie (2005)

Performance-avoidance:
Avoid getting one of the
lowest scores in the class.

Mastery-approach:
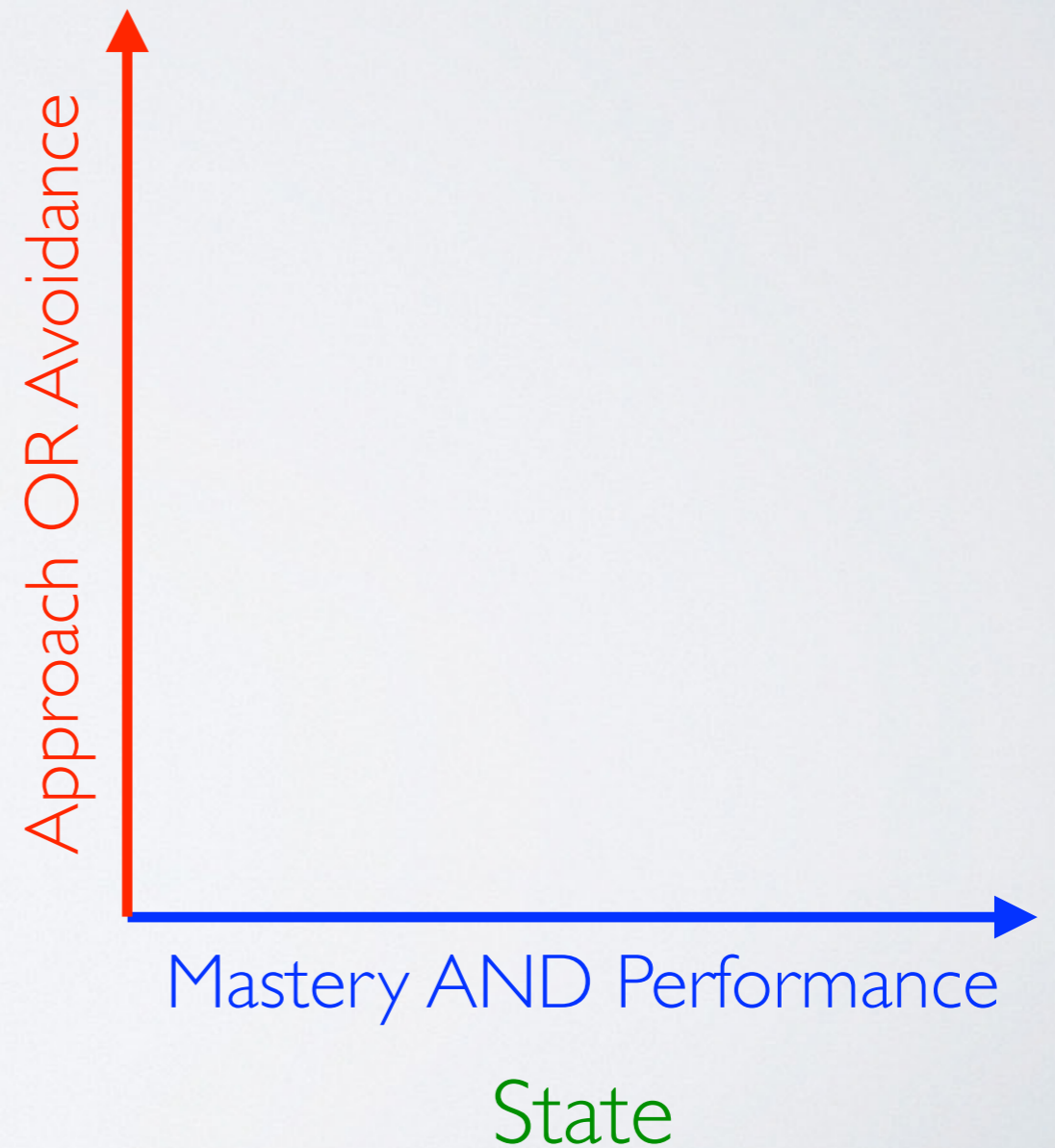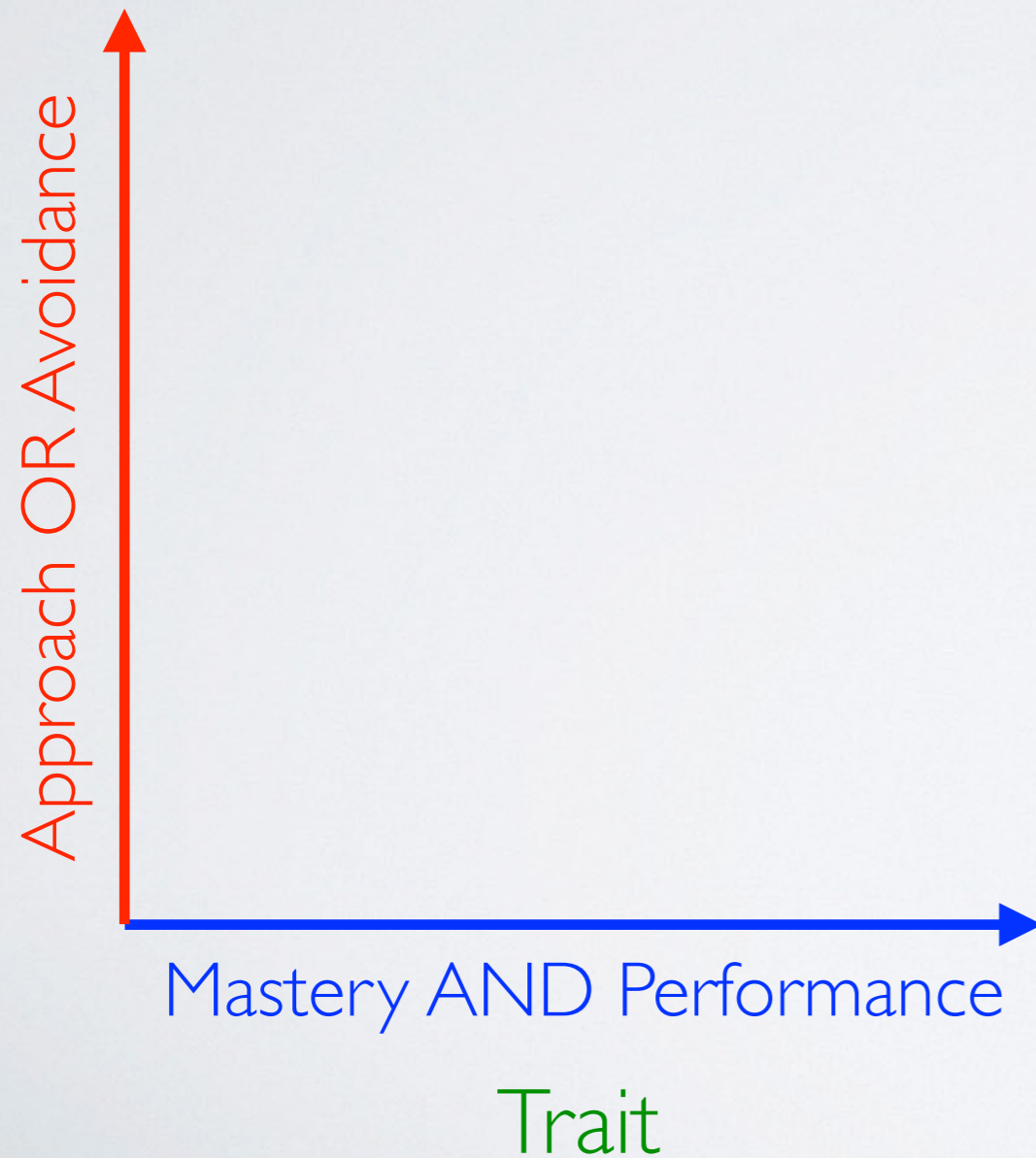Understand the idea of proof
by induction.

Approach OR Avoidance

Mastery AND Performance

GOAL ORIENTATION

Deshon and Gillespie (2005)

GOAL ORIENTATION

Deshon and Gillespie (2005)

Trait (left): vertical axis "Approach OR Avoidance", horizontal axis "Mastery AND Performance"

State (right): vertical axis "Approach OR Avoidance", horizontal axis "Mastery AND Performance"
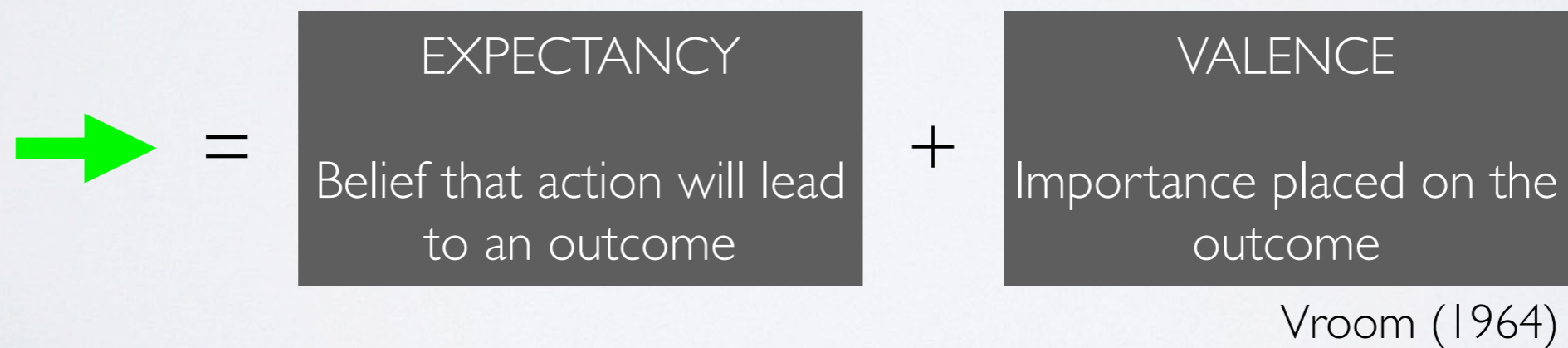
INCLUSIVE TEACHING

BELONGING

- Belonging

  ‣ "My teacher knows my name."

- Peer acceptance and support

  ‣ "[My groupmate] told me I did a good job… she acknowledged what I was good at."

- Instructor acceptance and support

  ‣ "[The professor] always tried to listen to your opinion and understand it."

INCLUSION ➔ ACHIEVEMENT

"[Inclusive] classroom environment perceptions predicted students' belonging beliefs, which in turn predicted students' motivation, engagement, and achievement in the course."

| Inclusive Classroom Environment | ➔ | Belonging | ➔ | Motivation | ➔ | Achievement |
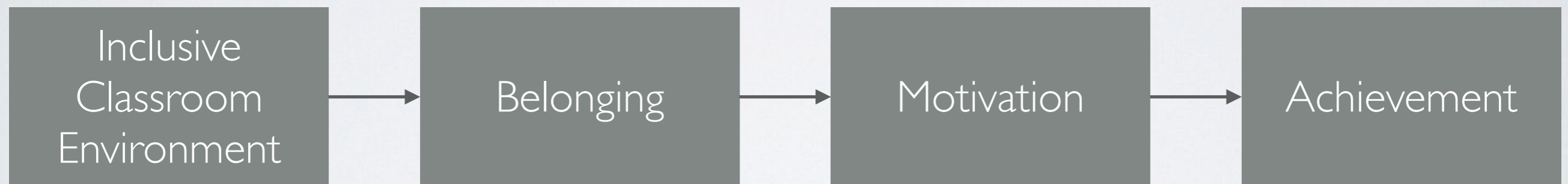
S. Zumbrunn, C. McKim, E. Buhs, L.R. Hawley. Support, belonging, motivation, and engagement in the college classroom: A mixed method study, *Instructional Science* 42 (5) (2014) 661-684.

STRATEGIES FOR INCLUSION

- Syllabus design

- Inclusive language

- Implicit bias mitigation

- Stereotype threat reduction

- Active learning

SYLLABUS DESIGN: WHAT TO DO

- Is explicit about everything

- Has fair policies

- Provides learning outcomes, prerequisites

- Explains assessment/evaluation scheme

- Serves as a resource
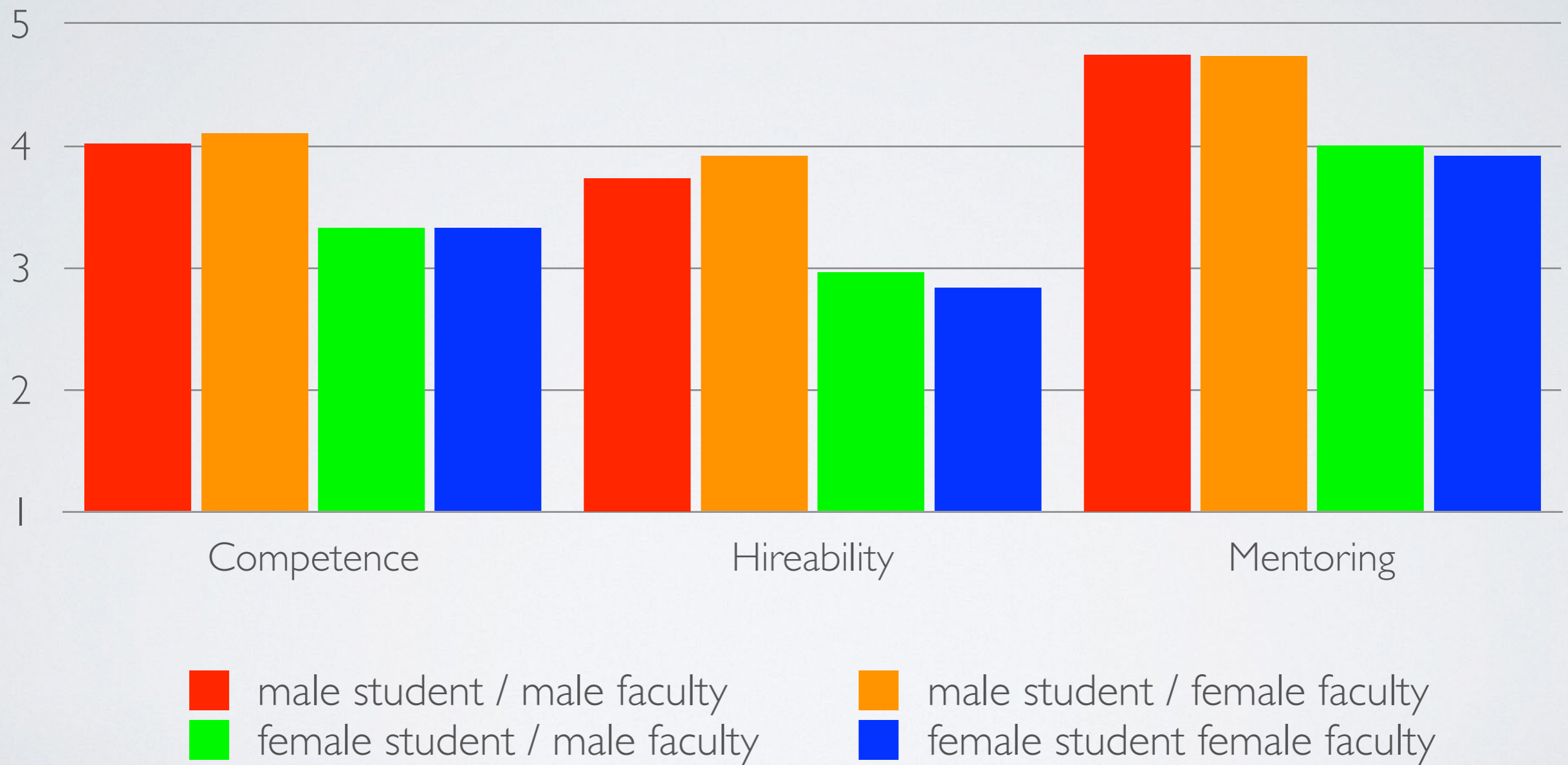
- Signals an inclusive environment

INCLUSIVE LANGUAGE:

- Give students (electronically) chance to tell pronouns

- When unsure, say "they" instead of "he/she"

- No more "you guys"

- Avoid "it's clear," "this one's easy," "obviously," etc.

- Talk about yourself as a person, talk about your struggles

IMPLICIT BIAS

- Unconscious attribution of particular qualities to a member of a certain social group

- Everyone has it

- Why? Brain is trying to shortcut

SCIENCE FACULTY'S SUBTLE GENDER BIASES FAVOR MALE STUDENTS

Legend:
- male student / male faculty (red)
- male student / female faculty (orange)
- female student / male faculty (green)
- female student female faculty (blue)

Moss-Racusin *et al.*, PNAS (2012)

COGNITIVE SOPHISTICATION DOES NOT ATTENUATE THE BIAS BLIND SPOT

"The so-called bias blind spot arises when people report that thinking biases are more prevalent in others than in themselves. In two studies, we found that none of these bias blind spots were attenuated by measures of cognitive sophistication. If anything, a larger bias blind spot was associated with higher cognitive ability."

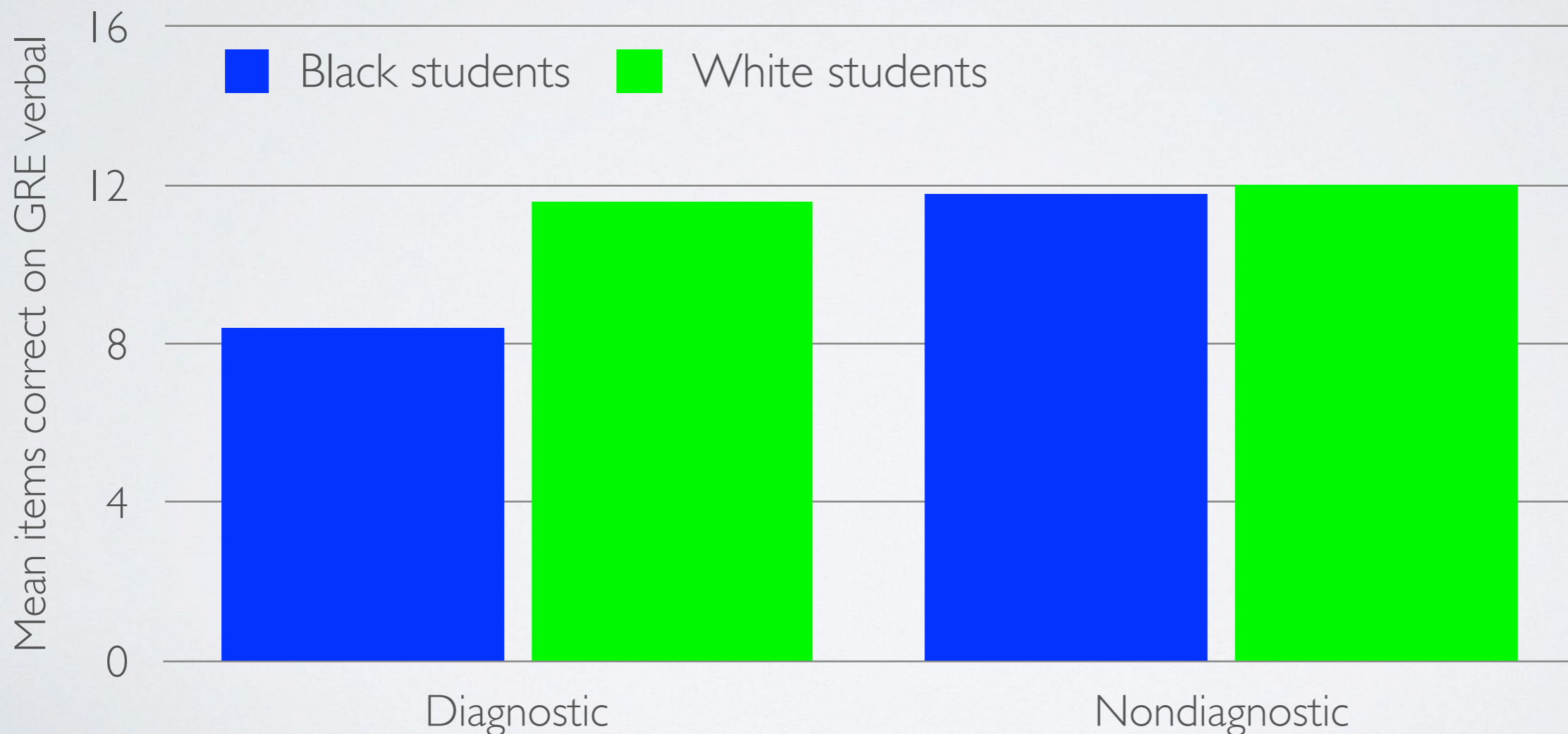West *et al.*, Person. Soc. Psych. (2012)

IMPLICIT BIAS: WHAT TO DO

- Build new associations via evaluative conditioning and counterstereotypical exemplars

- Solicit feedback from students, observers

- Slow down / practice mindfulness

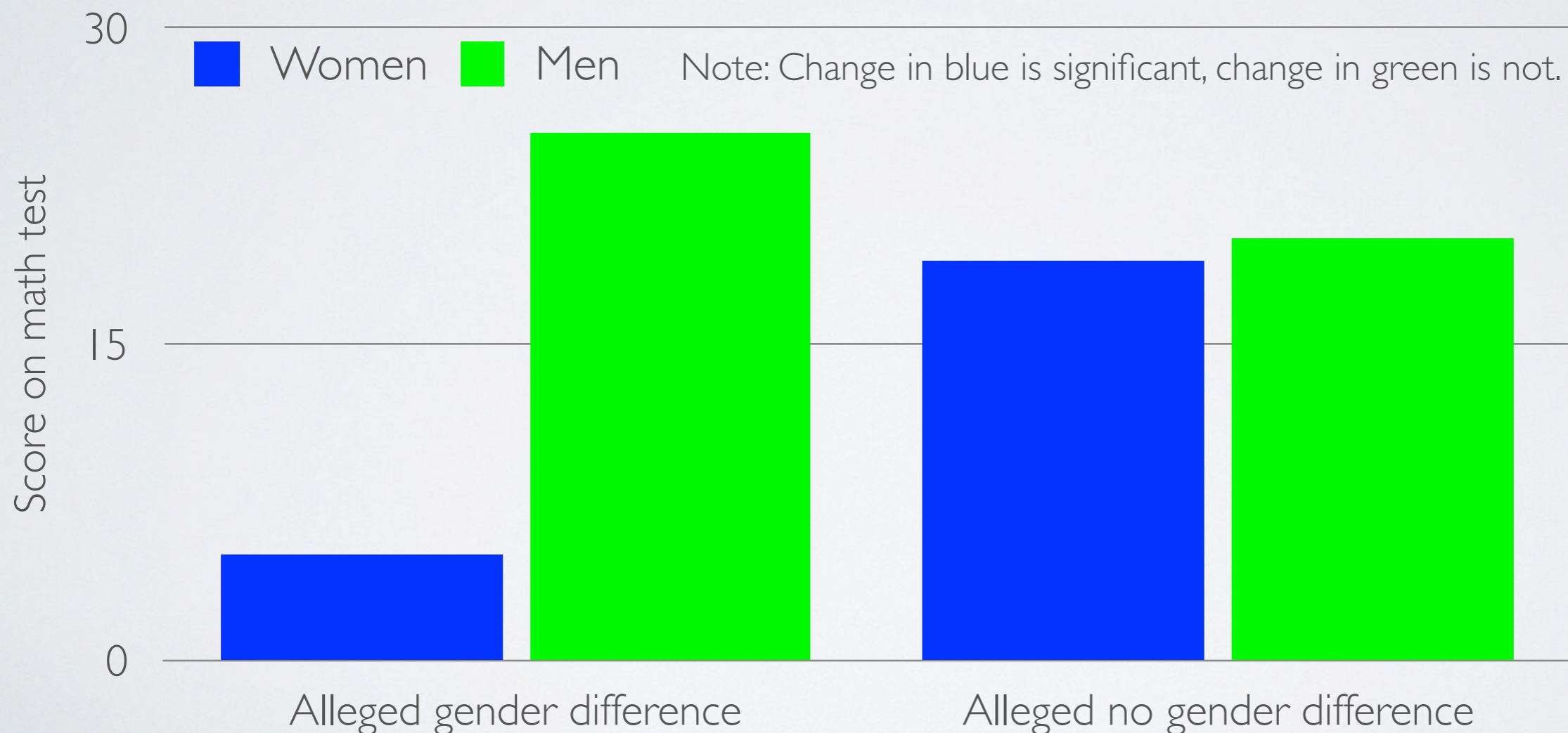- Enhance objectivity (*e.g.*, grade anonymously)

see, *e.g.*, Fitzgerald *et al.,* BMC Psych. (2019)

STEREOTYPE THREAT

- Feeling of risk of conforming to a stereotype

Steele and Aronson, J., Person. Soc. Psych. (1995)

STEREOTYPE THREAT

- Feeling of risk of conforming to a stereotype

Spencer, Steele, Quinn, J., Exp. Soc. Psych. (1999)

STEREOTYPE THREAT: WHAT TO DO

- Set high standards and communicate belief in students

- Have students affirm core values

- Increase representation of minority groups

- State that intelligence is malleable, achieved through effort

- Communicate that diversity is valued

- Construct environments and have the physical objects in the environment not reflect one majority group

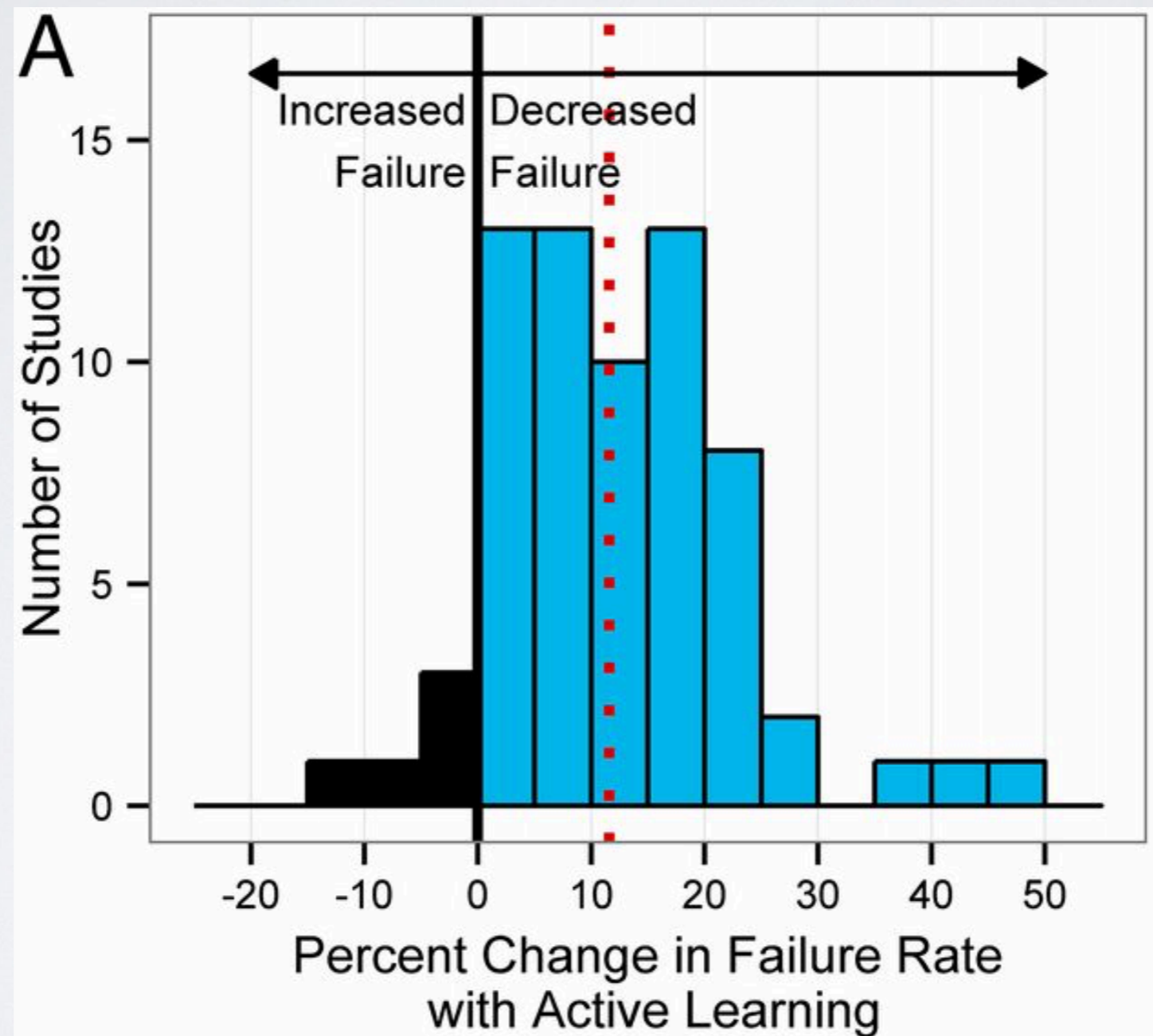https://en.wikipedia.org/wiki/Stereotype_threat#Mitigation

ACTIVE LEARNING
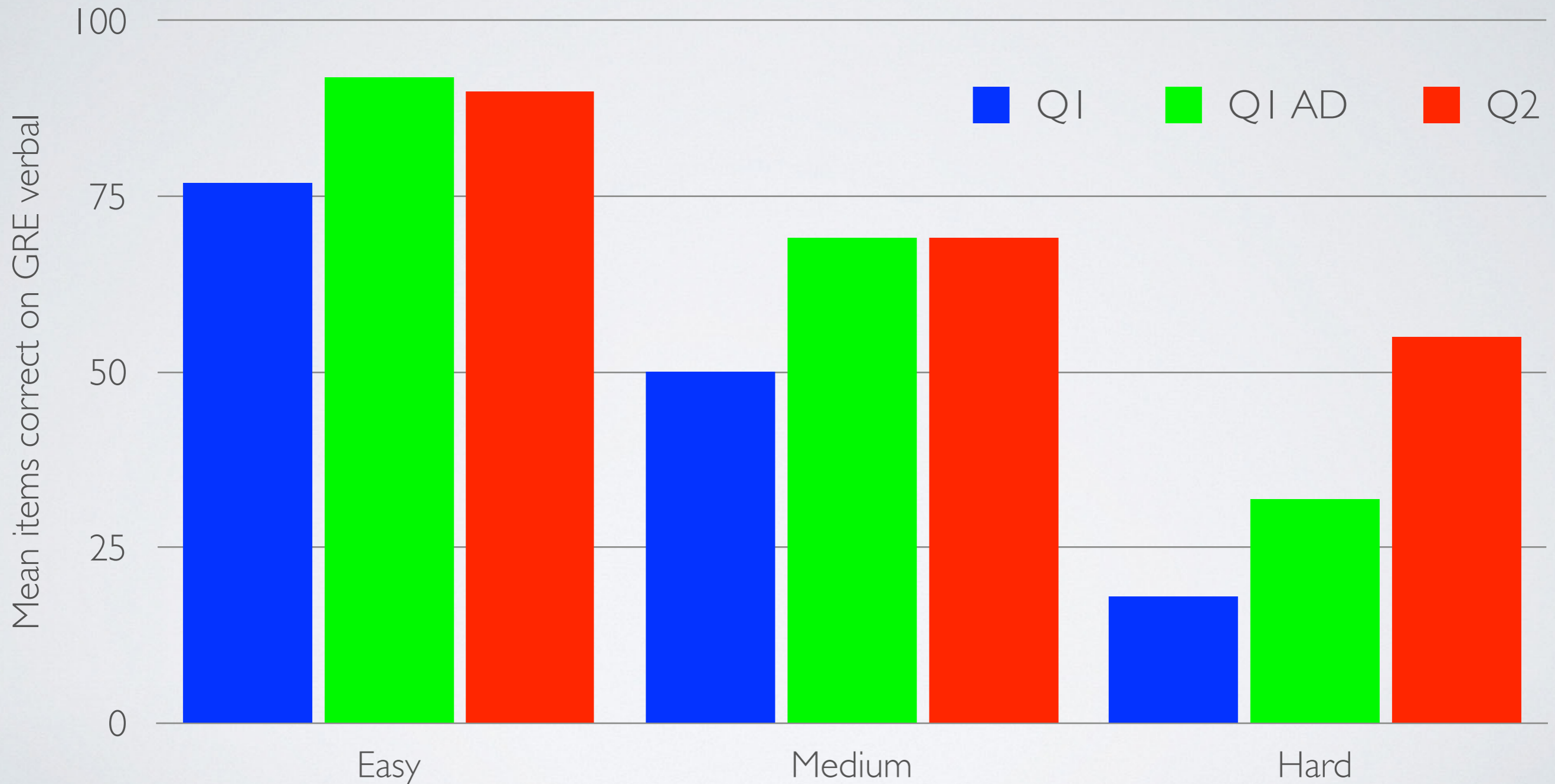
- What is active learning? Discuss.

ACTIVE LEARNING

- What is active learning? Discuss.

- When learners participate in the educational process by doing something besides listening

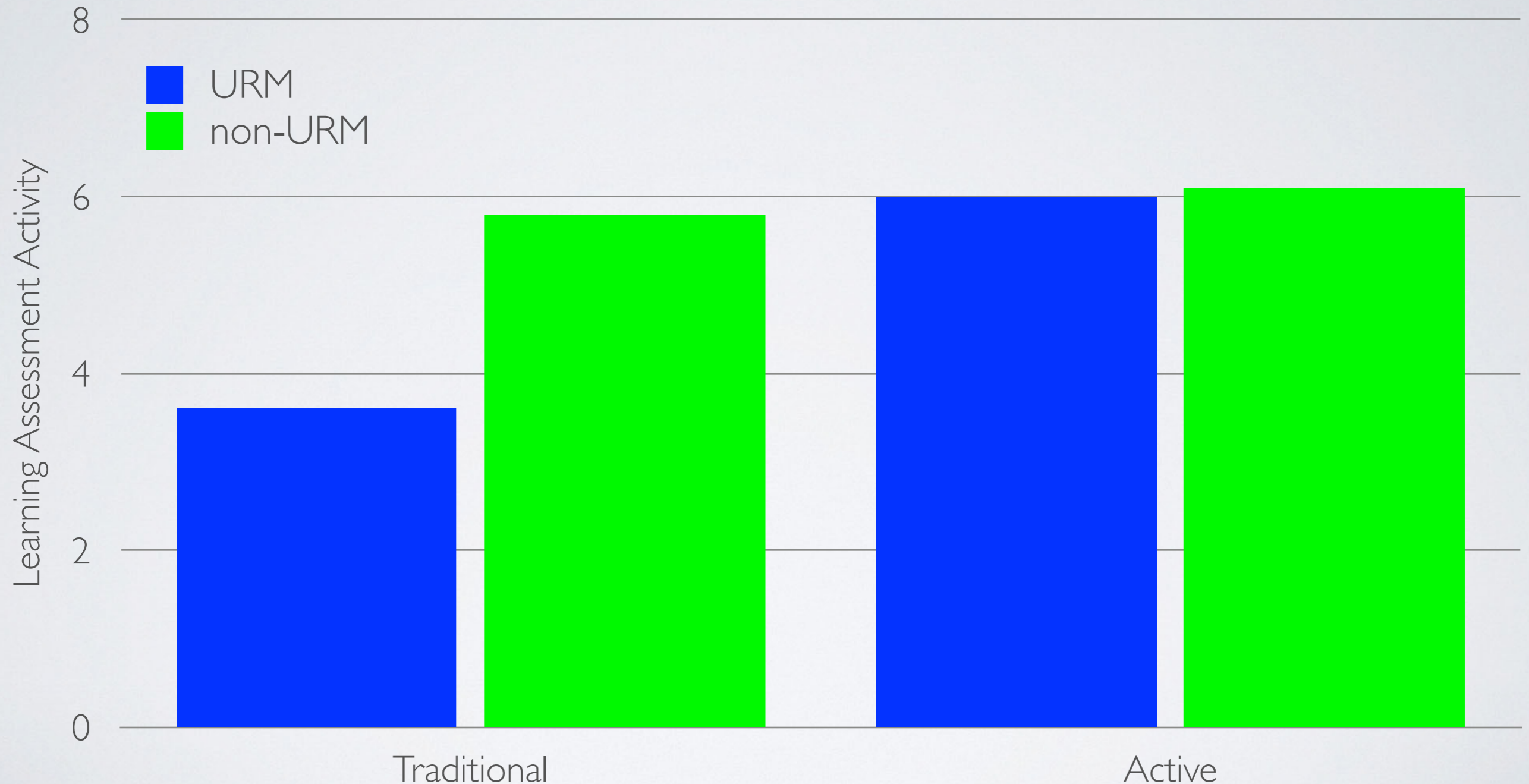# ACTIVE LEARNING INCREASES STUDENT PERFORMANCE IN STEM

Freeman *et al.*, PNAS (2014)

PEER DISCUSSION IMPROVES SCORES ON IN-CLASS CONCEPT QUESTIONS

Smith *et al.*, Science (2009)

ACTIVE LEARNING DECREASES THE PERFORMANCE GAP

Ballen *et al.*, CBE Life Sci. Ed. (2017)

ACTIVE LEARNING: WHAT TO DO

- Think/pair/share

- One minute muddiest point paper

- Concept questions / peer instruction

- Group activities (worksheets, labs, etc.)

- Individual + group quizzes

- Peer feedback (exchange solutions, etc.)

- Many more!

DISCRETE MATHEMATICS

COURSE DESCRIPTION

This course will help you understand the world through a mathematical lens and will develop your powers of argumentation and critical thinking. We will explore and utilize diverse areas of discrete mathematics including logic, set theory, functions and relations, combinatorics, probability, networks, and more. We also will discuss methods and styles of mathematical proofs in order to prepare you for more advanced math courses. Finally, while mathematical knowledge is often perceived as being "pure," we will highlight some ways in which it is socially constructed and hence subject to human limitations and biases.

COURSE FORMAT

To afford students flexibility during the COVID pandemic, this course is taught online. Students will watch lecture material asynchronously and will participate in a once-per-week synchronous small-group tutorial meeting with the instructor via video chat.

EVALUATION

Students will complete checkpoint quizzes on videos and reading assignments, regularly assigned homework problems, and reflective writing assignments. To move towards a non-hierarchical, transparent, and egalitarian grading system, the instructor follows the policy of "ungrading." Over the course of the semester, students will develop a rubric to assess their own learning and will evaluate themselves according to this rubric.

SYLLABUS

Discrete Mathematics

MATH 200
Fall 2020, Online Course

Instructor
Chad Topaz
cmt6@williams.edu

Teaching Assistants
TBA
TBA

TBA
TBA

Office hours
Times TBD
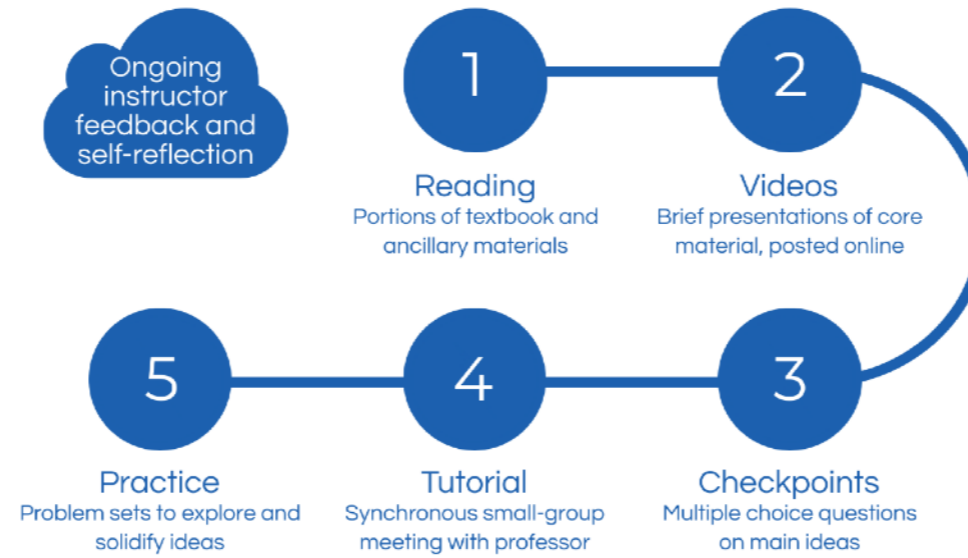Online (Zoom)

Help sessions
Times TBD
Online (Zoom)

COURSE DESCRIPTION

Understand the world through a mathematical lens and increase your powers of argumentation and critical thinking! We explore and utilize diverse areas of discrete math including logic, set theory, functions, relations, combinatorics, probability, networks, and more.

SYLLABUS

CLASS STRUCTURE

Ongoing instructor feedback and self-reflection

1 Reading
Portions of textbook and ancillary materials

2 Videos
Brief presentations of core material, posted online

3 Checkpoints
Multiple choice questions on main ideas

4 Tutorial
Synchronous small-group meeting with professor

5 Practice
Problem sets to explore and solidify ideas

SYLLABUS

COURSE ENVIRONMENT

Names/Pronouns
You deserve to be addressed in the manner you prefer. To guarantee that I address you properly, you are welcome to tell me your pronoun(s) and/or preferred name at any time, either in person or via email.

Diversity
We embrace diversity of gender, gender expression, sex, sexual orientation, race, ethnicity, national origin, age, religion, disability status, family status, socioeconomic background, and other visible and non-visible categories. I do not tolerate discrimination.

Accessibility
I want you to succeed in this course. Contact glw3@williams.edu for learning accommodations. For personal issues, stress, health problems or life circumstances, contact the Dean's office at x4171. Contact me if you have other special circumstances. I will find resources for you.

Title IX
You deserve a community free from discrimination, sexual harassment, a hostile environment, sexual assault, domestic violence, dating violence, and stalking. If you experience or know of a Title IX violation, you have many options for support and/or reporting; see titleix.williams.edu.

Academic Integrity
The honor code is a cornerstone of our learning community and of this course. It is your responsibility to know and follow academic integrity policies. I will gladly answer any questions you have.
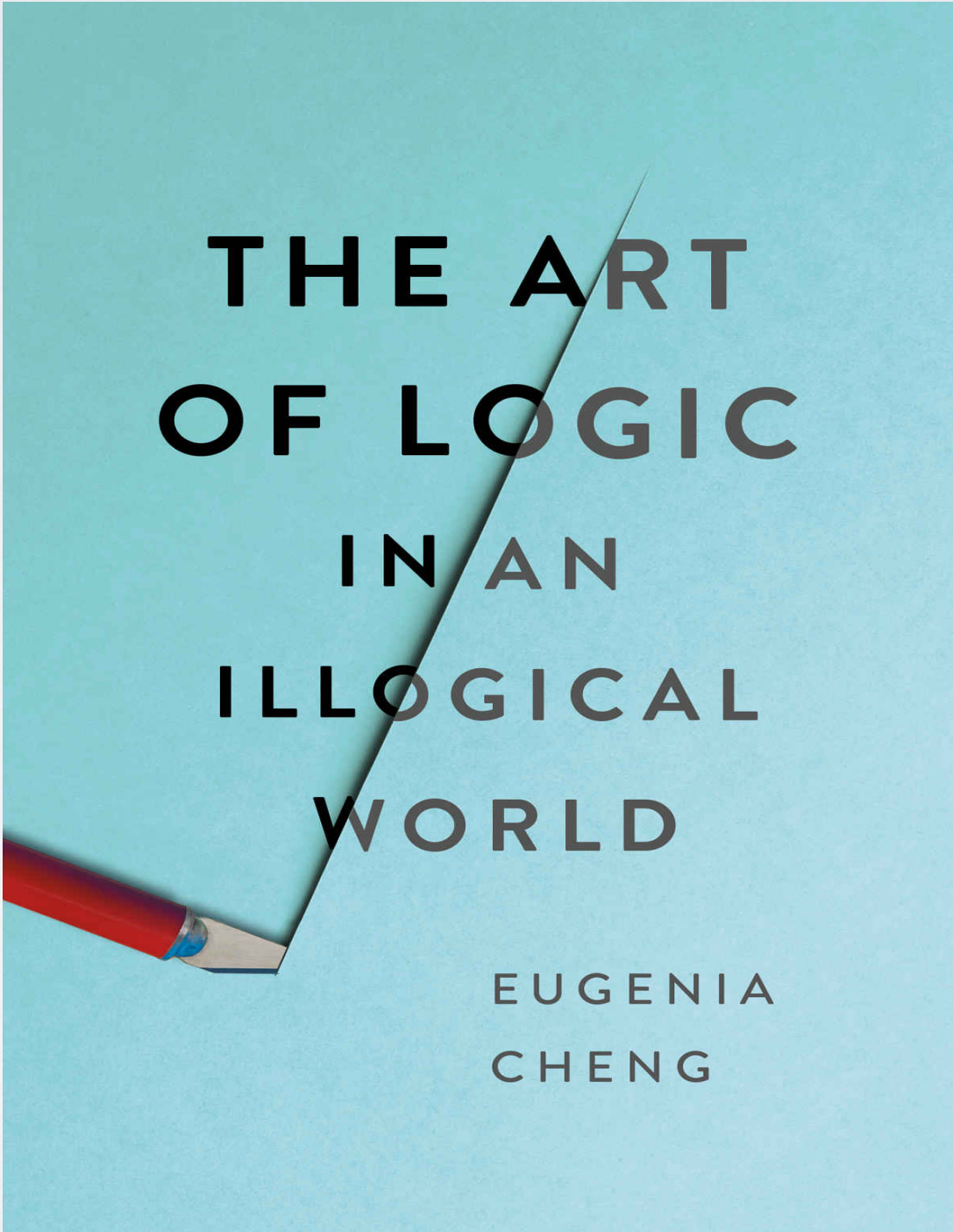
PEDAGOGICAL DESIGN

	Reading	Videos	Checkpts	Tutorial	Problem Sets	Asynch. Discuss.	Office Hours	Self Eval.
Knowledge Centered	X	X	X	X	X	X	X	X
Learner Centered		X		X		X	X	X
Assessment Centered			X	X	X		X	X
Community Centered				X	X	X		

READING
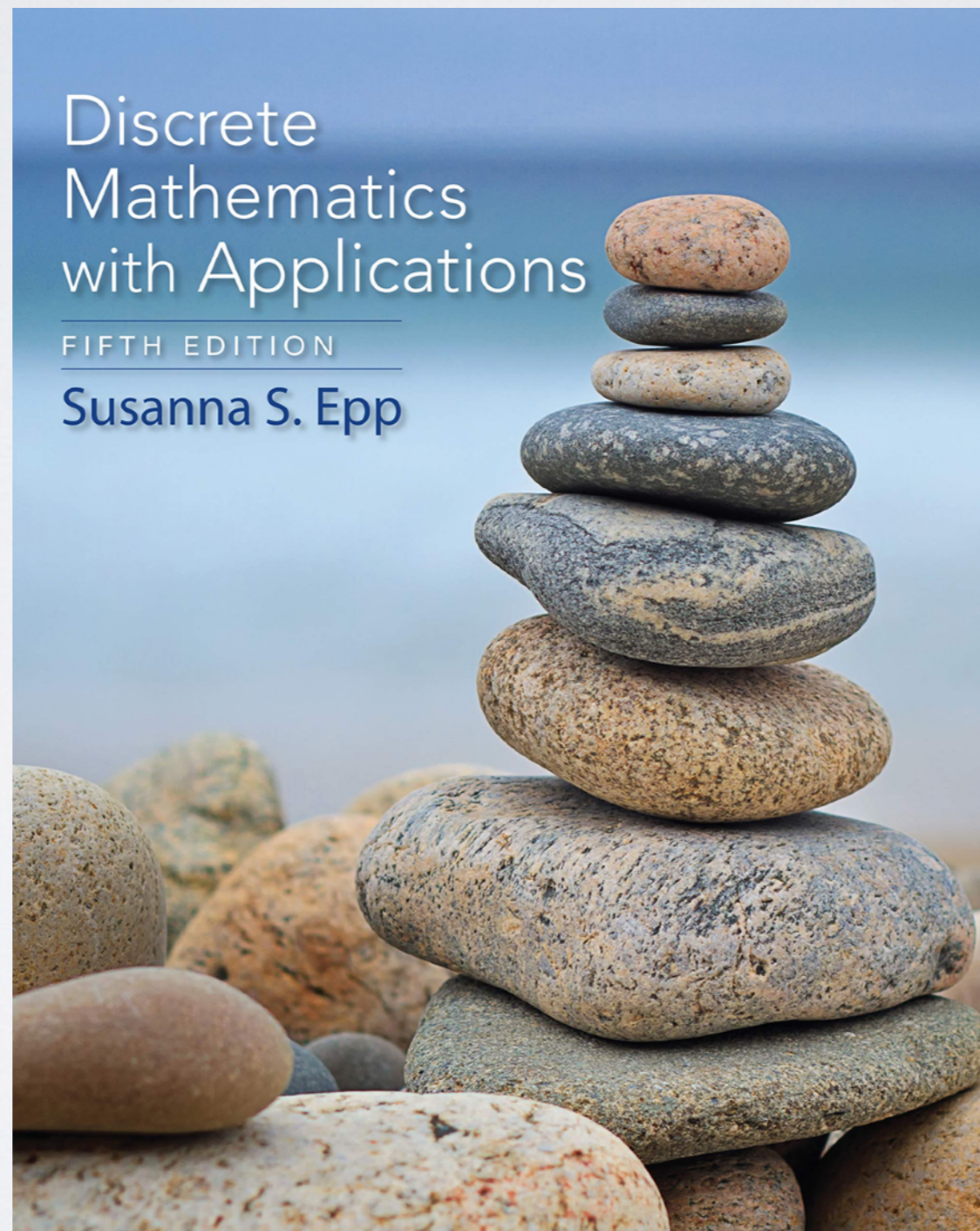
READING

THE ART
OF LOGIC

IN AN

ILLOGICAL

WORLD

EUGENIA
CHENG

READING

A HISTORY OF INDIAN LOGIC

(ANCIENT, MEDIAEVAL AND
MODERN SCHOOLS.)

BY

Mahāmahopādhyāya SATIS CHANDRA VIDYABHUSANA,
M.A., Ph.D., M.R.A.S., F.A.S.B.,

*Principal Sanskrit College, Calcutta; Joint Philological Secretary
Asiatic Society of Bengal, Fellow of the Calcutta
University; etc.*

MOTILAL BANARSIDASS
DELHI :: PATNA :: VARANASI

Diversity in Proof Appraisal

Matthew Inglis and Andrew Aberdein

1 Proof Appraisals

A clichéd view of research-level mathematics, or at least research-level pure mathematics, is that it is simply and solely concerned with logic: purported proofs are either valid or invalid, and the job of a mathematician is to produce as many valid ones as possible. On this account, there is little place for the appraisal of proofs in anything other than a straightforwardly descriptive fashion. Proofs might be valid or invalid, published or unpublished, short or long, but under the clichéd view it is hard to see how they could be elegant, beautiful, or deep. However, a cursory glance at mathematical practice reveals that mathematicians regularly make such appraisals. For example, in the citation for the 2003 Abel Prize, Jean-Pierre Serre's work was described as being "profound", "spectacular", and "magnificent". So it is clear that a mathematician's appraisal of a given piece of work can go well beyond its validity. In this chapter we specifically focus on the evaluation of mathematical proofs: how are such appraisals made, and what is their status?

A more systematic investigation of the ways in which mathematicians characterise mathematical proofs is given in Table 1. It shows the most common adjectives used to characterise proofs on MathOverflow, a website where research

M. Inglis (✉)
Mathematics Education Centre, Loughborough University, Leicestershire, UK
e-mail: m.j.inglis@lboro.ac.uk

A. Aberdein
School of Arts and Communication, Florida Institute of Technology, Melbourne, USA
e-mail: aberdein@fit.edu

Epistemic injustice in mathematics

Colin Jakob Rittberg[1] · Fenner Stanley Tanswell[2] ·
Jean Paul Van Bendegem[1]

Abstract
We investigate how *epistemic injustice* can manifest itself in mathematical practices. We do this as both a social epistemological and virtue-theoretic investigation of mathematical practices. We delineate the concept both positively—we show that a certain type of folk theorem can be a source of epistemic injustice in mathematics—and negatively by exploring cases where the obstacles to participation in a mathematical practice do not amount to epistemic injustice. Having explored what epistemic injustice in mathematics can amount to, we use the concept to highlight a potential danger of intellectual enculturation.

Keywords Epistemic injustice · Virtues · Philosophy of mathematics · Folk theorems · Caramello · Royen · Ramanujan · Enculturation

1 Introduction

Desirable mathematics extends beyond the confines of correct calculations. Desirable mathematics has to do with the behaviour of the building blocks of mathematics: we value *deep* theorems, *beautiful* proofs, *fruitful* theories, and so on.[1] Because mathematics is done by human agents, desirable mathematics also has to do with the ways in which mathematicians perform their craft: we value traits such as creativity, metic-

---

[1] See e.g. Maddy (2011, 2017) for *depth* and *fruitfulness* and Rota (1997) for *beauty*.

✉ Colin Jakob Rittberg
colin.jakob.rittberg@vub.be

Fenner Stanley Tanswell
Fenner.Tanswell@gmail.com

Jean Paul Van Bendegem
jean.paul.van.bendegem@vub.be

[1] Vrije Universiteit Brussel, Brussels, Belgium

[2] Mathematics Education, Centre Loughborough University, Loughborough, UK

Springer

READING

For practice

Rewrite the statement formally using quantifiers and variables. Then, write a negation for each statement.

- Everybody loves somebody.

- Somebody loves everybody.

- Any integer equals twice some integer.

- There is a program that gives the correct answer to any question that is posed to it.

Section 3.4: Arguments with quantified statements

Instantiation

- The **rule of universal instantiation** says that if some property is true of *everything* in a set, then it is true of *any particular* thing in the set.

- For example, Williams students work too hard. Anita is a Williams student. Therefore, Anita works too hard.
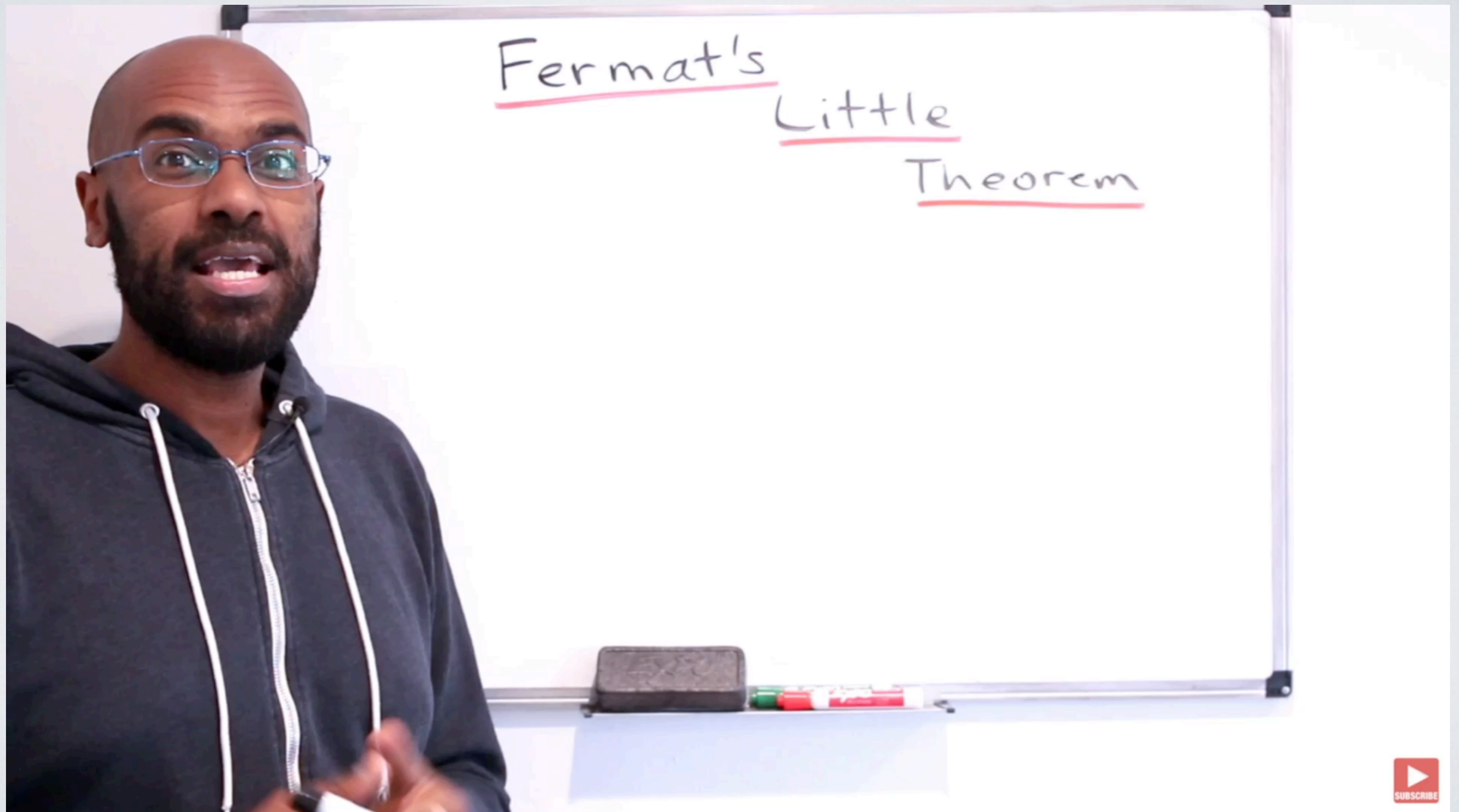
Universal Modus Ponens

- Universal instantiation can be combined with *Modus Ponens* to construct an argument called **universal Modus Ponens**:

  - $\forall x$ if $P(x)$ then $Q(x)$.
  - $P(a)$ for a particular a.
  - $\therefore Q(a)$

- For example, if T is any right triangle with hypotenuse c and legs a and b, then $c^2 = a^2 + b^2$. A given triangle is a right triangle with both legs equal to 1 and hypotenuse c. Then $c^2 = 1^2 + 1^2 = 2$.

Universal Modus Tollens

- Similarly, niversal instantiation can be combined with *Modus Tollens* to construct an argument called **universal Modus Tollens**:

  - $\forall x$ if $P(x)$ then $Q(x)$.
  - $\sim Q(a)$ for a particular a.
  - $\therefore \sim P(a)$.

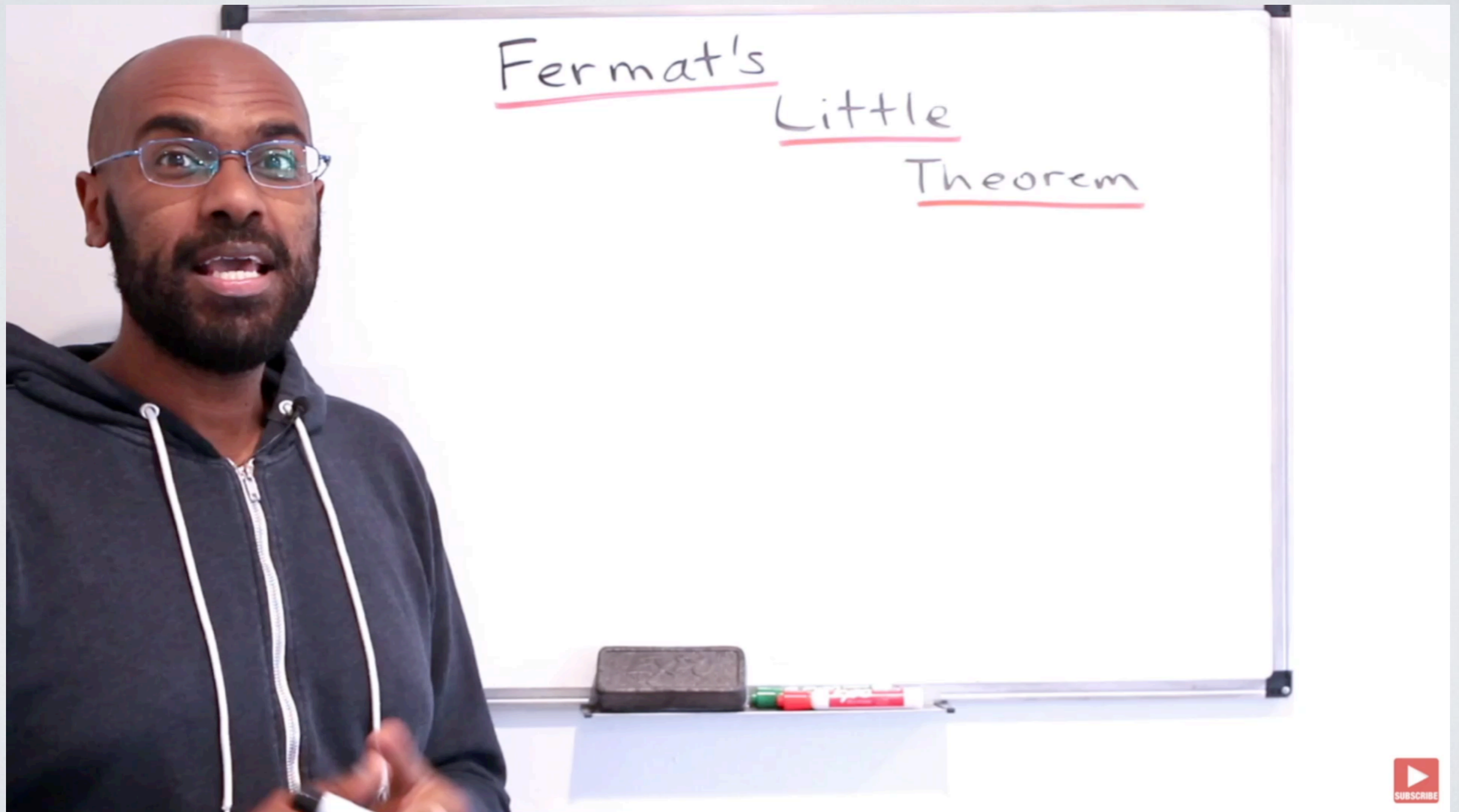- For example, all professors are absent-minded. Jude is not absent-minded. Therefore Jude is not a professor.
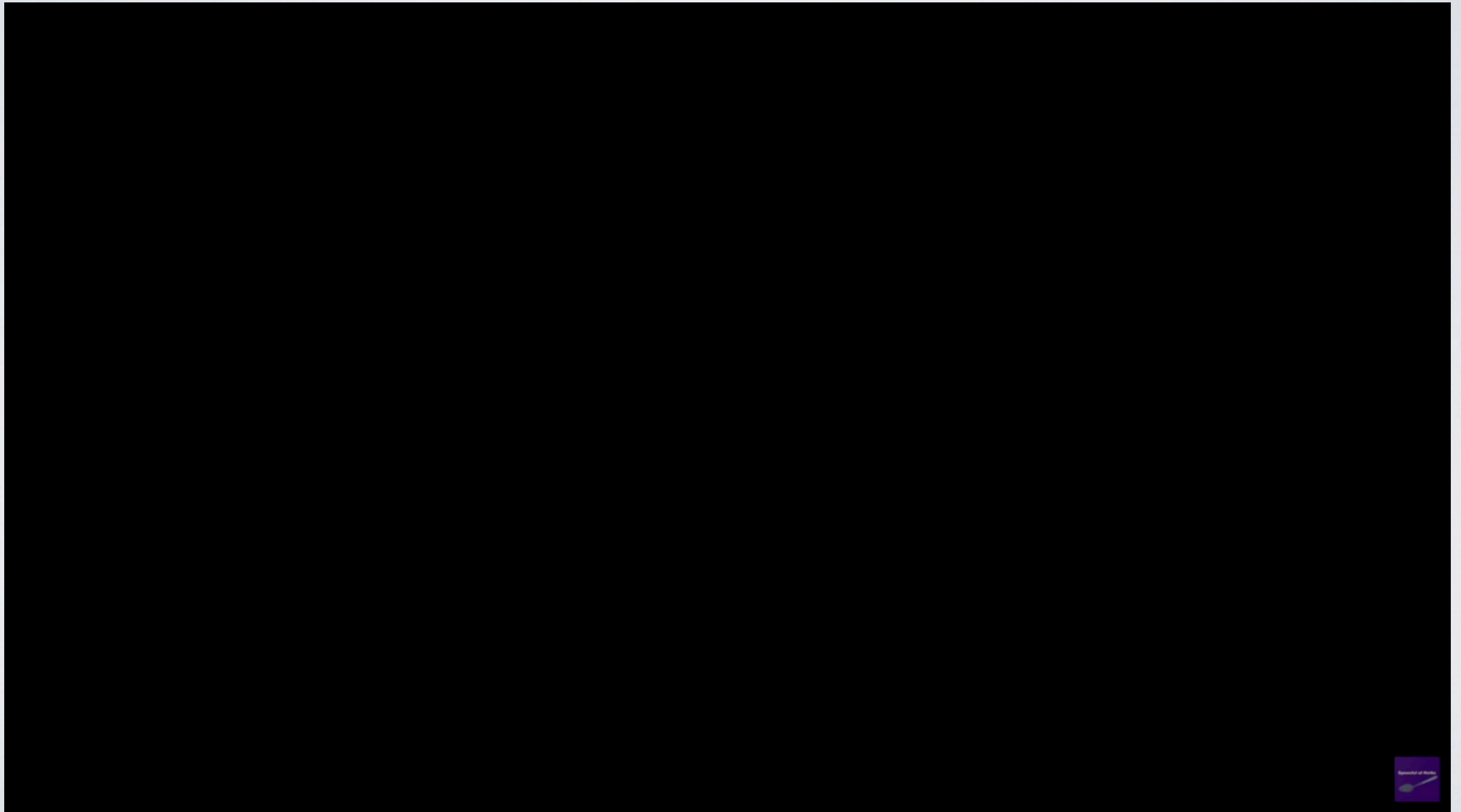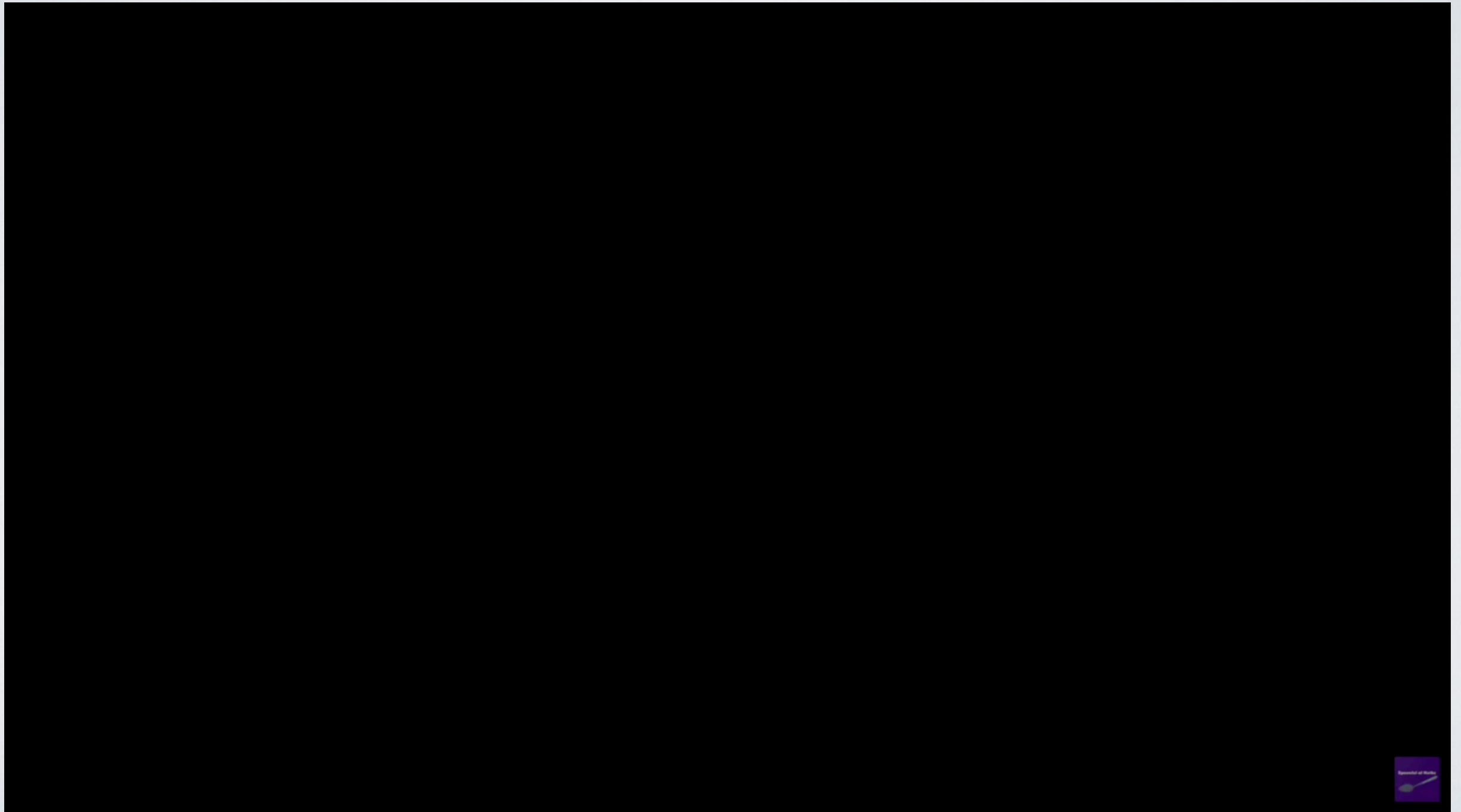
VIDEOS

Prof. Mo Omar (https://www.youtube.com/watch?v=zP9t00lPXiU)

VIDEOS

Prof. Mo Omar (https://www.youtube.com/watch?v=zP9t001PXiU)

VIDEOS

Dr. Sara Herke (https://www.youtube.com/watch?v=qZ736F8ljYU)

VIDEOS

Dr. Sara Herke (https://www.youtube.com/watch?v=qZ736F8IjYU)

CHECKPOINTS

1. The statement p ∨ (p ∧ q) is logically equivalent to p ∧ (p ∨ q).
   - True
   - False

2. The negation of "they tested positive for COVID and they are not infected" is…
   - They didn't test positive for COVID and they are infected
   - They didn't test positive for COVID or they are infected
   - They didn't test positive for COVID or they are not infected
   - They tested positive for COVID and they are infected
   - They tested positive for COVID or they are not infected

TUTORIAL

ZOOM

Prof.

Student 1

Student 2

Student 3
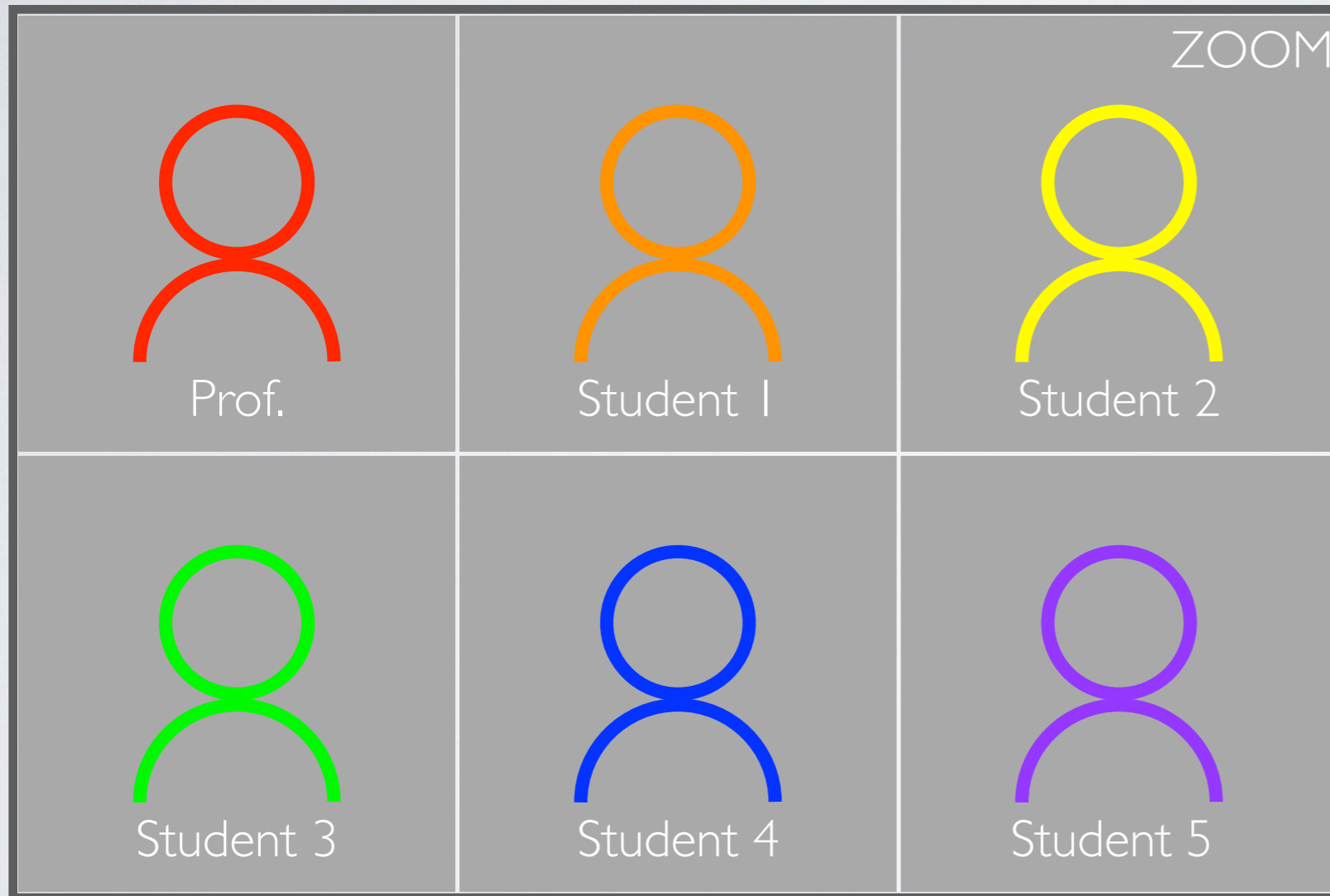
Student 4

Student 5

- 75 min 2x/week
- 2 sections
- 30 students/section
- 5 students/tutorial

- 300 minutes
- 12 tutorials

- 25 min tutorials

WEEKLY SELF-EVALUATION

1. Why are you taking this course? What are your goals for yourself?

2. Articulate a plan for how you will "do" this course. In your plan, please include how much time you will allocate to this course each week, what you will do when you encounter challenges, and so forth. Be as specific and granular as you can.

3. Keeping in mind your goals for taking this course, what criteria/metrics/experiences/reflections will you use to evaluate yourself?

4. Now, using the criteria you discussed above, please evaluate your work in this course so far.

FINAL REFLECTIONS ON MY APPROACH

- Being human is more important than doing math.

- An online course is, first and foremost, a course.

- The learning science literature is valuable.

- You don't have to do All The Things.