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Re: American Mathematical Society Response to 88 FR 26345, Request for Information on Developing a Roadmap for the Directorate for Technology, Innovation, and Partnerships at the National Science Foundation

Comments transmitted electronically via *TIPRoadmap-RFI@nsf.gov* on July 20, 2023

Dear Dr. Baru,

The American Mathematical Society (AMS) appreciates the opportunity to provide comments in response to the Request for Information (RFI) on Developing a Roadmap for the Directorate for Technology, Innovation, and Partnerships at the National Science Foundation (88 FR 26345). Founded in 1888, the AMS is dedicated to advancing the interests of mathematical research and scholarship and connecting the diverse global mathematics community. The AMS has approximately 30,000 individual members worldwide and supports mathematical scientists at every career stage.

The National Science Foundation (NSF) Technology, Innovation, and Partnerships (TIP) Directorate is critical to advancing US competitiveness in science and technology while also simultaneously addressing societal challenges. There is a clear need for increased investment in use-inspired, solution-oriented research that impacts people's everyday lives. While mathematics and statistics are foundational and often curiosity-driven, they also play a key role in use-inspired research. Advancing theoretical foundations in the mathematical sciences has led to extraordinary applications—but that is often not the whole story. Practical applications can lead to novel mathematical and statistical theories, which give rise to further applications. And the cycle of translational mathematical sciences research continues from there.

As such, the deployment of state-of-the-art mathematical sciences research and equitable and inclusive access to high-impact mathematical sciences education should be incorporated in TIP's roadmap, and its ten key technology and five societal, national, and geostrategic focus areas. Mathematical scientists, both researchers and educators, should be consulted and play an active role in convenings, funding opportunities, review panels, and advisory committees. To accomplish this, we recommend that TIP 1) offer specific "on ramps" for the use of mathematical sciences in TIP activities and 2) use several NSF programs as "pit stops" in its roadmap.

The mathematical sciences are integral to understanding, modeling, and solving complex problems across the broad spectrum of technologies. Here, we share examples of how the mathematical sciences can be deployed in use-inspired research in TIP's ten key technology areas.

- <u>Artificial intelligence, machine learning, autonomy, and related advances:</u> Numerous subfields of mathematics and statistics are theoretical foundations of AI and machine learning. For example, various optimization algorithms, such as gradient-based methods, evolutionary algorithms, and convex optimization, are used to train machine learning models and tune (hyper)parameters.

  It is important to understand what is or is not reasonable for AI to accomplish, not just to understand the potential of large language models or for general AI but also to address concerns about robustness in the models and protecting privacy (e.g., preventing a user of the model from recovering specific training data items by querying the model). Current approaches to these questions, sometimes categorized as "machine learning security," use high-dimensional geometry and optimization.

  In the "thinking fast and slow" dichotomy, neural networks think fast. TIP's technology roadmap should also include the computational analogue of thinking slowly, namely theorem-proving systems such as Lean[1] and Coq[2]. Humans think in both styles, and it is natural to believe that a future, more powerful form of AI will incorporate also some tools from the thinking slow side.

- <u>High performance computing, semiconductors, and advanced computer hardware and software:</u> High performance computing routinely uses the mathematical sciences for performance, simulations, and analyses. For example, high performance computers rely on numerical methods and optimization algorithms for analyzing large data sets and for complex simulations (e.g., weather and climate forecasting).

  For semiconductors, multiscale modeling is currently being used for radiation hardening (i.e., making electronic components and circuits resistant to radiation damage). Most semiconductors are susceptible to radiation damage and reliability is critical in extreme environments as found in outer space and nuclear facilities.

- <u>Quantum information science and technology:</u> The mathematical formalism of quantum mechanics, including complex vector spaces, linear algebra, and operators, provides the language to describe quantum systems, their states, and their evolution

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[1] https://leanprover.github.io/
[2] https://coq.inria.fr/

over time. Mathematical concepts such as superposition and entanglement are central to quantum information processing.

TIP's technology roadmap should include continued development of the mathematical tools that underpin quantum-secure cryptography (i.e., cryptographic systems that cannot be compromised by a quantum computer).[3] Including these topics will help maintain the security and integrity of all modern electronic communications, ranging from the internet to security updates for apps to satellite communications.

It is important to investigate verification and validation of quantum computing hardware. For producing integrated circuits, extensive manufacturing quality assurance techniques have been developed, but our understanding of analogous processes for quantum computing hardware are less developed.

- <u>Robotics, automation, and advanced manufacturing:</u> Mathematics is used to determine the position, orientation, and movement of robotics in advanced manufacturing. Algorithms such as inverse kinematics and trajectory planning use mathematical principles to enable precise control of robotic systems.[4, 5]

- <u>Natural and anthropogenic disaster prevention or mitigation:</u> Weather forecasting has dramatically improved over the past decade because of advances in theoretical mathematics and statistics, innovative mathematical and statistical models, constantly improving approaches toward integrating data, efficient computing, and expanded data collection. Accelerated changes in climate necessitate further improvements in data collection and integration and models.[6]

  TIP's technology roadmap should include the study of probability and stochastic processes, partial differential equation, scientific computing, model reduction, rare events, risk management, and statistical and mathematical modeling. Including these topics will contribute to the understanding and management of events and phenomena as varied, for example, as economic and financial disasters, diseases and pandemics, extreme weather events, and collapses of infrastructure, and deliver improvements in areas such as healthcare delivery, design and simulation of materials, and resilience of telecommunications networks, battery and energy technology.

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[3] https://www.nist.gov/news-events/news/2022/07/nist-announces-first-four-quantum-resistant-cryptographic-algorithms
[4] https://nap.nationalacademies.org/catalog/25481/data-driven-modeling-for-additive-manufacturing-of-metals-proceedings-of
[5] https://nap.nationalacademies.org/resource/other/deps/illustrating-math/interactive/manufacturing-with-mathematics.html
[6] https://nap.nationalacademies.org/resource/other/deps/illustrating-math/interactive/mathematics-and-statistics-of-weather-forecasting.htmlTIP's

- Advanced communications technology and immersive technology: Coding theory centers on the design and analysis of error-correcting codes that help ensure accurate transmission of information over noisy channels. Mathematical concepts such as algebraic structures (e.g., finite groups, fields, rings and others), linear codes, and combinatorics are used to develop efficient error-correcting codes.

  Quantum-secure cryptography is crucial for secure data transmission, authentication, and digital signatures. Mathematical concepts such as algebraic structures and probabilistic and statistical tools (e.g., rejection sampling) are used to develop and certify practical quantum-secure cryptographic constructions.[7]

- Biotechnology, medical technology, genomics, and synthetic biology: Despite conditions of uncertainty and variability, living systems are maintained with a high degree of fidelity, while being able to vary, adapt, and evolve in a complex and changing environment.

  TIP's technology roadmap should foster advances in dynamical systems, topology, and probability theory to understand and mimic some of the most efficient structures present in nature. Including these topics will lead to effective designs of synthetic organisms, the development of more cost-effective drugs and the discovery of the next generation of medical treatments. Notably, mathematical and computational modeling can be combined with knowledge of key steps in development to inform design of artificial tissues and organs.[8]

- Data storage, data management, distributed ledger technologies, and cybersecurity, including biometrics: TIP's roadmap should include topics such as integer lattices, error-correcting codes, and isogenies of super-singular elliptic curves. Research in these areas is intimately connected with future cryptographic systems that may prove to be secure against both classical and quantum computers.[9] Some uses of these cryptographic systems are further topics of fundamental research, such as zero-knowledge proofs, secure multiparty computation, and fully homomorphic encryption, which will rely on these post-quantum primitives in order to be secure against classical and quantum computers. Real-world uses of zero-knowledge proofs, for example, is still evolving, with perhaps the most prominent current use in distributed ledger technologies and verifiable voting systems such as Microsoft's ElectionGuard[10]. Progress on these topics will also help the Federal Reserve to successfully deploy a central bank digital currency.[11]

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[7] https://nap.nationalacademies.org/resource/other/deps/illustrating-math/interactive/mathematics-of-internet-security.html

[8] https://www.whitehouse.gov/wp-content/uploads/2023/03/Bold-Goals-for-U.S.-Biotechnology-and-Biomanufacturing-Harnessing-Research-and-Development-To-Further-Societal-Goals-FINAL.pdf

[9] https://csrc.nist.gov/Projects/Post-Quantum-Cryptography

[10] https://news.microsoft.com/on-the-issues/2020/03/27/what-is-electionguard/

[11] https://www.federalreserve.gov/cbdc-faqs

- Advanced energy and industrial efficiency technologies, such as batteries and advanced nuclear technologies, including but not limited to for the purposes of electric generation: Advanced energy technologies often require optimization and control techniques to maximize energy generation, minimize costs, and improve efficiency. Mathematical optimization methods—including linear and nonlinear programming and optimal control theory—are applied to determine best possible operation strategies, scheduling of energy resources, and control policies for energy systems. These techniques help optimize the allocation of resources, predict energy demand, and manage the integration of intermittent renewable sources into the grid.[12]

- Advanced materials science, including composites 2D materials, other next-generation materials, and related manufacturing technologies: Mathematics is used to describe crystal structures and study their symmetries, which have a direct influence on the properties of materials. Mathematical tools (e.g., statistical mechanics) are often applied to understand the electronic behavior and thermal properties of solids.[13]

  TIP's technology roadmap should include nanoscale materials characterization using multiresolution mathematical analysis and synthesis tools—such as numerical simulations of nanostructures and theoretical formulations. These advances are underpinning the development of materials with unique and novel properties, such as invisibility or metamaterial cloaking, new optical characteristics, lighter weight, and increased strength. Emphasizing mathematical aspects in materials science and engineering will help advance the innovation, discovery, and fabrication of next generation materials to address pressing needs in communications, clean energy, medicine, and national security.

Turning to the use of the mathematical sciences for addressing societal, national, and geostrategic challenges, we highlight a few examples of the deep impact.

- United States national security: The mathematical sciences are actively used for national security across the US military, for example, in Bayesian target tracking or sensor fusion. The Department of Defense funds research in mathematics and statistics through their research arms and through DARPA. The US National Security Agency is perhaps the most prominent example of the importance of mathematics for national security, since it is the nation's largest single employer of mathematicians and employs over 600 PhD mathematicians and partners with hundreds of mathematicians at federally funded research and development centers.

  TIP's roadmap should address training the next generation of US citizen mathematicians so that the Department of Defense has the talent they need in order

[12] https://nap.nationalacademies.org/catalog/21919/analytic-research-foundations-for-the-next-generation-electric-grid
[13] https://nap.nationalacademies.org/catalog/26723/nsf-efforts-to-achieve-the-nations-vision-for-the-materials-genome-initiative

to secure the nation. For instance, TIP could investigate a partnership with the National Defense Science and Engineering Graduate Fellowship Program.

- United States manufacturing and industrial productivity: TIP's roadmap should address theoretical modeling of robotics including dynamical systems and control theory, logistic and optimization, graph theory and network modeling and analysis. Addressing these topics will facilitate the automatization of function in the manufacturing process increasing the US competitiveness in terms of speed of fabrication, effective distribution in the supply chain distribution of goods, and robust resilient networks.[14]

- United States workforce development and skills gaps: Skills in mathematical sciences empower and inspire. Our STEM workforce in particular must routinely use a wide range of mathematical and statistical skills. Students and workers need strong foundations in mathematical sciences to advance the modern science, technology, and engineering opportunities.

  TIP's roadmap should include educational and training programs on topics in the mathematical sciences that reinforce transferable skills in logical thinking and the ability to analyze problems, encourage exploration and discovery, and highlight the applicability of those topics to a broad range of social challenges.

- Climate change and environmental sustainability: TIP's roadmap should address statistical and stochastic modeling, machine learning, scientific computing, and uncertainty quantification. Addressing these topics will improve the reliability and value of climate models, enable more accurate and actionable predictions at smaller spatiotemporal scales than are currently possible, and improve the modeling of interactions between climate and biological, ecological, and sociological systems.

- Inequitable access to education, opportunity, or other services: Access to high-quality mathematics education is paramount for the US prosperity and global leadership in STEM. For example, access to Algebra II courses is an important bellwether for students attaining STEM degrees and pursuing careers. And yet access to Algebra II is not universal across the United States. TIPS's roadmap should address outreach and education efforts for K-12 and undergraduate students, as well as efforts to attract and train teachers so they can teach Algebra II, calculus, and elementary probability and statistics. The evolving changes in all aspects of technology will require harnessing the skills and talents of the whole population, bringing in as many varied perspectives as possible, and this cannot be achieved without access to advanced mathematics for all students.

TIP has a new and profoundly forceful role for NSF: scaling up the things that work. Accordingly, TIP's roadmap should include pit stops at NSF programs and events to learn

[14] https://nap.nationalacademies.org/catalog/26989/autonomous-materials-discovery-and-optimization-proceedings-of-a-workshop-in

and expand success stories. We share several NSF activities that TIP should support at scale.

- Transdisciplinary Research In Principles of Data Science: The TRIPODS program aims to advance the foundations of data science by supporting collaborative research projects that integrate principles from mathematics, statistics, theoretical computer science, and other relevant disciplines. The program encourages transdisciplinary approaches to address the challenges posed by large-scale, complex datasets and extract meaningful insights from them.

  TIP can foster communications between three communities (mathematics, statistics, computer science) and local industry, facilitate collaboration, and establish partnerships. In addition, TIP can enhance student training through TRIPODS programs, by expanding the traditional mode of in-school-only training to academic-industry co-training to better prepare future data scientist for solving real-world problems.

  Borrowing from the TRIPODS model and in appropriate solicitations, TIP should explicitly suggest or require mathematicians and statisticians to be incorporated in PI teams.

- NSF Research Traineeship: The NRT program aims to promote the development of innovative and interdisciplinary graduate training programs. The program seeks to train future scientists and engineers in cutting-edge research areas while also equipping them with professional skills to succeed in a variety of career paths.

  TIP should scale up internship programs and lab placements for PhD students, collaborative programs with industry, and summer schools on a broad range of mathematical science topics of relevance to TIP's areas of interest.

- Robert Noyce Teacher Scholarship Program: The Noyce Program provides scholarships, stipends, and professional development opportunities to undergraduate STEM majors, graduate students, and professionals transitioning to teaching careers. The program supports these individuals as they pursue degrees and certification in STEM fields with the intention of becoming K-12 teachers.

  TIP should work closely with the Directorate for STEM Education to build on the success of the Noyce program. Importantly, TIP should address both the recruitment and retention of educators in the mathematical sciences. For example, TIP should help current educators connect with industry and non-profits for professional development and student resources. Meanwhile, TIP should help build clear pathways for those in industry to transition to schools, colleges, and universities.

- Mathematical Sciences Institutes: The Mathematical Sciences Institutes are collaborative research centers focused on advancing mathematical sciences through research, training, and outreach activities. These institutes bring together mathematicians, statisticians, and researchers from related disciplines to foster innovation, collaboration, and the development of new mathematical knowledge.

TIP should scale up programs that enable and/or amplify research, training, and outreach activities at institutes that have interfaces with the technologies and challenges described above. The Simons Foundation's Targeted Grants to Institutes program provides one model which is worthy of consideration. It will be especially productive to work with institutes on activities which are organized around specific challenges or application areas.

We thank the NSF for the opportunity to provide these suggestions to TIP. If you have any questions or you would to discuss these suggestions in more detail, please do not hesitate to contact me and the AMS.

Kind regards,

Dr. Bryna Kra
President, American Mathematical Society